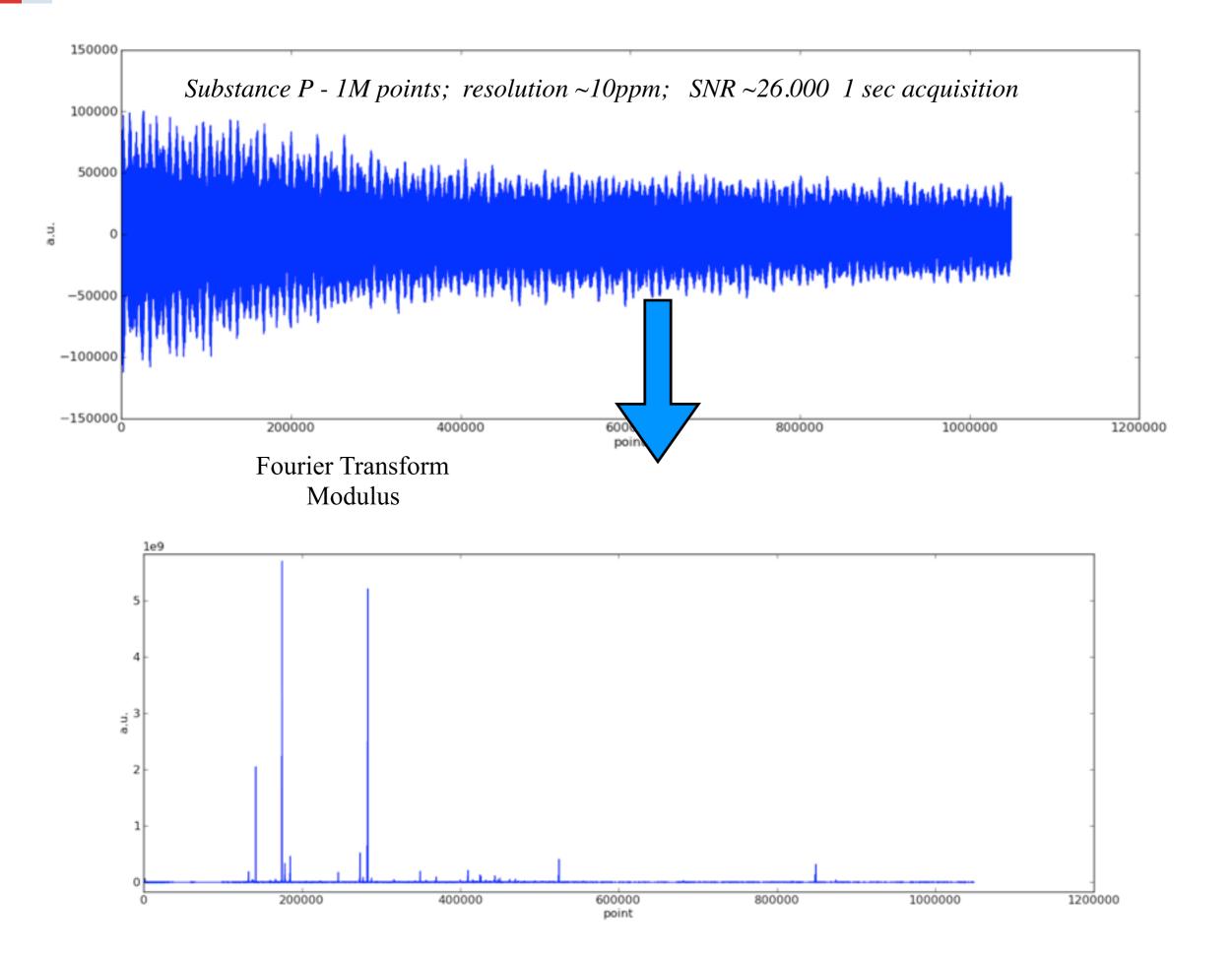
Extensions aux approches par transformée de Fourier

Marc-André Delsuc IGBMC

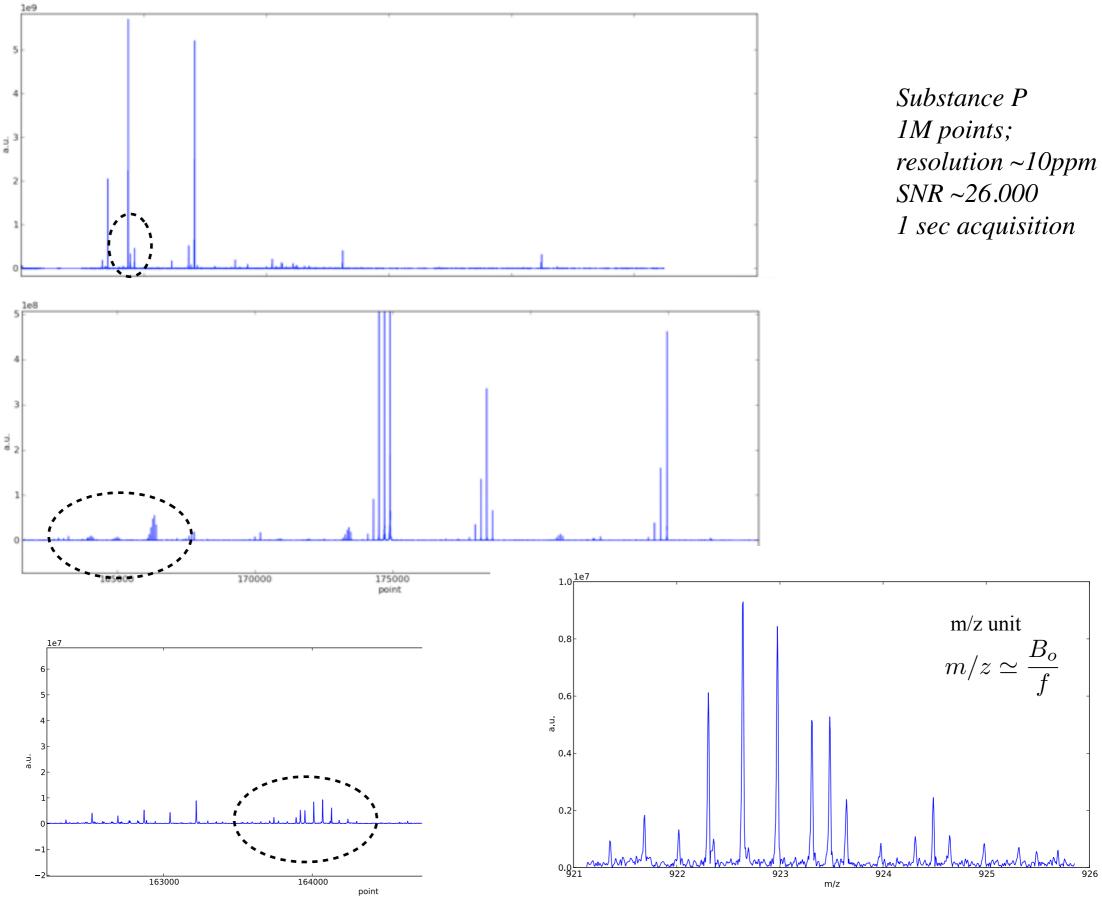






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Zooming in



Quel traitement de données ?

- Approche générale :
 - N points de mesures
 - P paramètres à extraire
- N > P
 - l'ajustement de paramètres (le *fit*)
 - <u>modélisation</u> du phénomène
- N = P
 - les **transformations** de données $T_f()$ inversible ou non
 - <u>modélisation</u> de la mesure
- N < P
 - la **reconstruction** de données
 - modélisation de la mesure et de la connaissance

MaxEnt Compressed-Sensing M-A Delsuc Ecole FTMS avril 2014

Zero-Filling X •débruitage

•2D

•Sur-résolution

 $y^{mes} = T_f(s) + \epsilon$

N

FDM

sur-resolution

• La résolution de la mesure est dominée par la relation de Gabor-Heisenberg

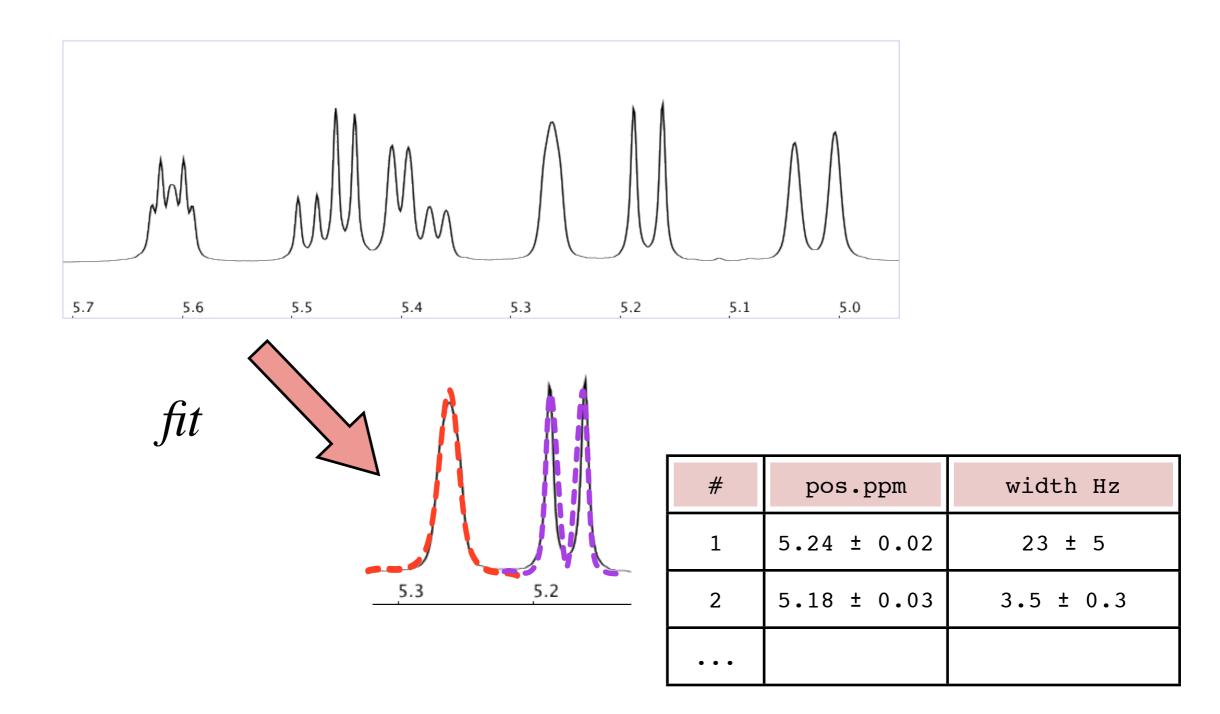
 $t_{max}\Delta F = 1$

maximiser la résolution pour une acquisition donnée

Trois approches

- - ajustement
- transformation
- reconstruction / régularisation
- dépend du rapport signal à bruit

N>P L'ajustement



N>P L'ajustement

modélisation du phénomène

- modèle physique du phénomène étudié
- on «ajuste» les P paramètres du modèle aux données mesurées
- écart mesuré par maximum de vraisemblance solution des «moindres carrés»

$$y_i^{calc} = f(P_j)$$

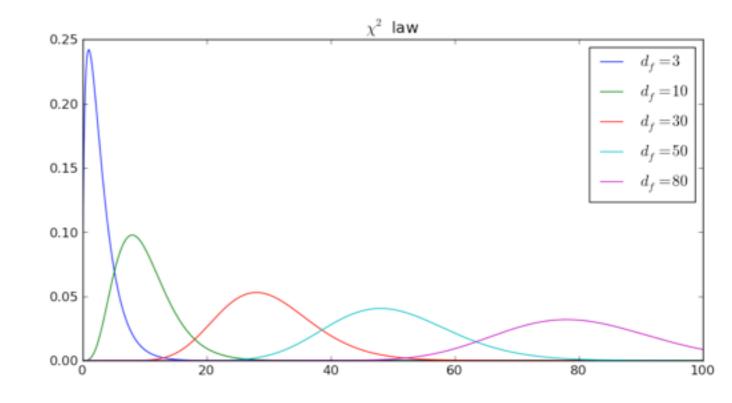
$$\chi^2 = \sum_i \left(\frac{y_i^{calc} - y_i^{mes}}{\sigma_i}\right)^2$$

degrés de liberté

• df = N-P

Loi du Chi2

- moyenne = df
- maximum = df-2
- variance = 2df



/

Principes

• minimiser le Chi2

- c'est une fonction convexe/quadratique \Rightarrow convergence rapide
- $f(P_i)$ peut-être complexe \Rightarrow minimum non-unique
 - \Rightarrow importance du choix des valeurs initiales
- Dans la pratique
 - utiliser une bibliothèque toute faite !

curve_fit(f, xdata, ydata, **kw[, p0, sigma]) Use non-linear least squares to fit a function, f, to data.

 pour chaque paramètre ajusté on peut estimer l'erreur à partir de la matrice de covariance

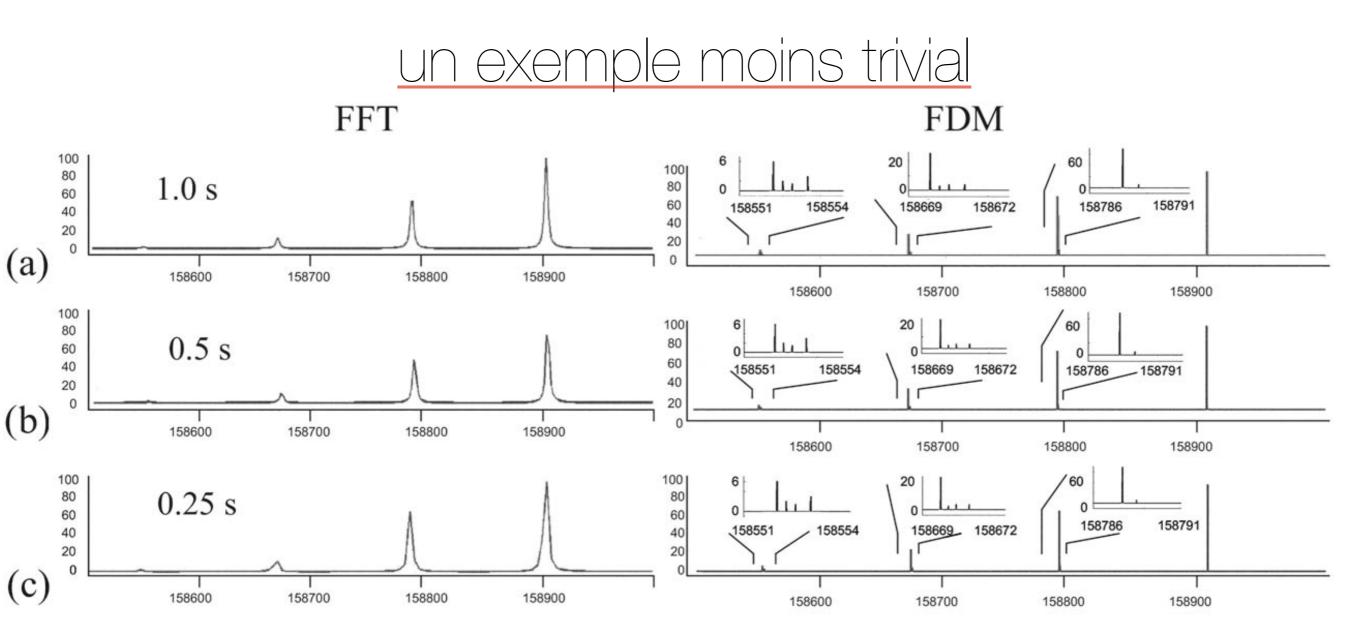
Returns

popt : array

Optimal values for the parameters so that the sum of the squared error of f (xdata, *popt) - ydata is minimized

pcov : 2d array

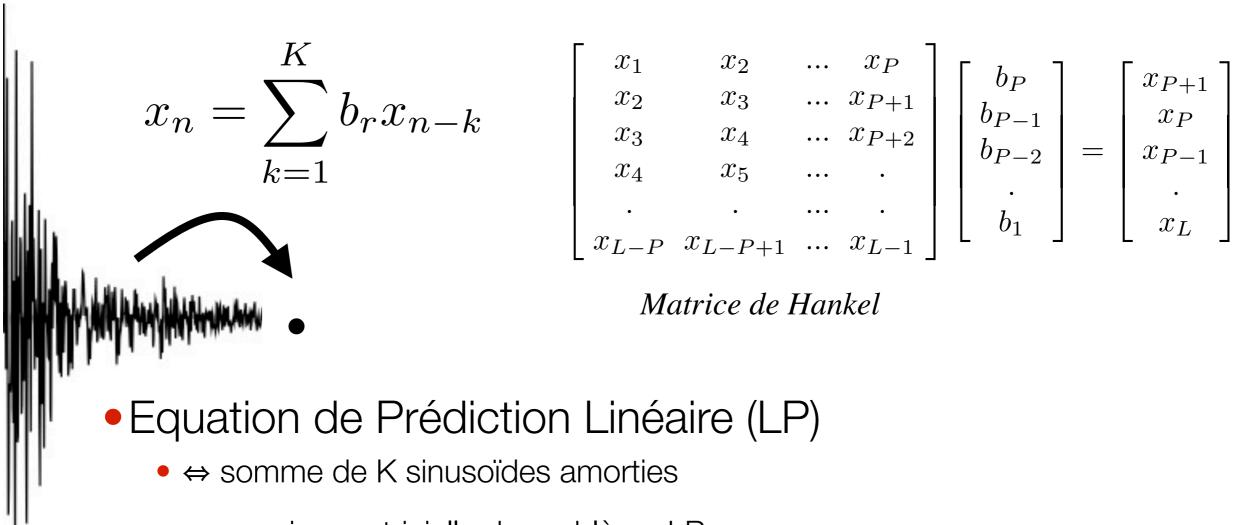
The estimated covariance of popt. The diagonals provide the variance of the parameter estimate.



Aizikov et O'Connor. J.Am. Soc.Mass Spec. (2006) 17 (6) pp. 836-843

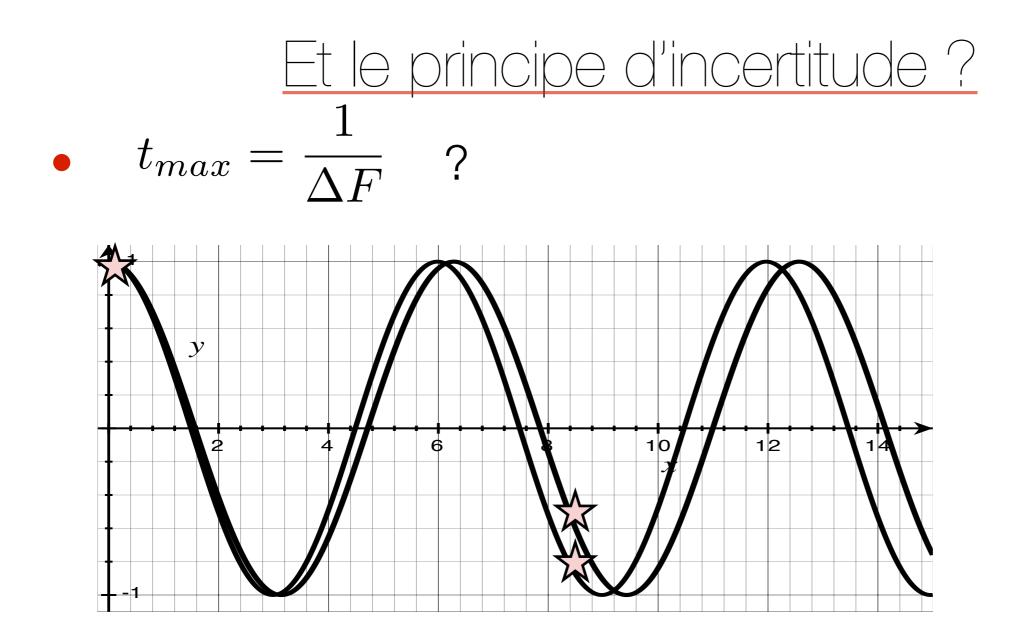
- Modèle AR : autoregressif
 - signal : somme de **n sinusoides** amorties **exponentiel**lement
- Filter Diagonalization Method
 - equivalent à un fit dans les données temporelles
 - méthode intelligente pour choisir les valeurs initiales; convergence optimisée

Modèle AutoRegressif



- expression matricielle du problème LP Hb = x
- H est décomposée pour extraire les fréquences valeurs propres, valeurs singulières,
- Les intensités sont mesurées dans un deuxième temps par un moindre carré standard (Méthode de Prony)

Prony, R. Essai Expérimental et Analytique... J. de l'Ecole Polytechnique (Paris) 1, 24–76 (1795).



- modélisation du signal
 - nombre de signaux
 - forme des signaux
- Information *a-priori*

N=P la notion de transformée

changement de point de vue

 Typique des spectroscopies par FT (mais plus..) spectroscopies : NMR, FT-ICR, FT-IR, ... images : IRM, jpeg

$$y^{mes} = T_f(s) + \epsilon$$

• On **modèle** le processus de mesure

notions de spectre / de fonction de mesure notion de fonction d'instrument (réponse impulsionelle - points manquants) -... *hypothèse linéaire*

- le problème au moindre carré
 - trouver ~s le plus proche possible de s tel que
 - la solution «triviale» est ou n'est pas la solution

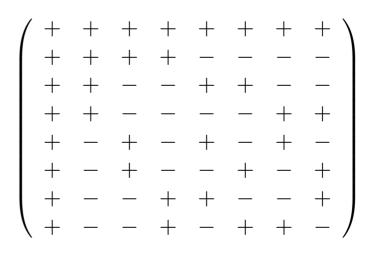
$$\chi^2 = \sum \left(\frac{T_f(s) - y^{mes}}{\sigma}\right)^2$$

$$\tilde{s} = T_f^{-1}(y^{mes})$$

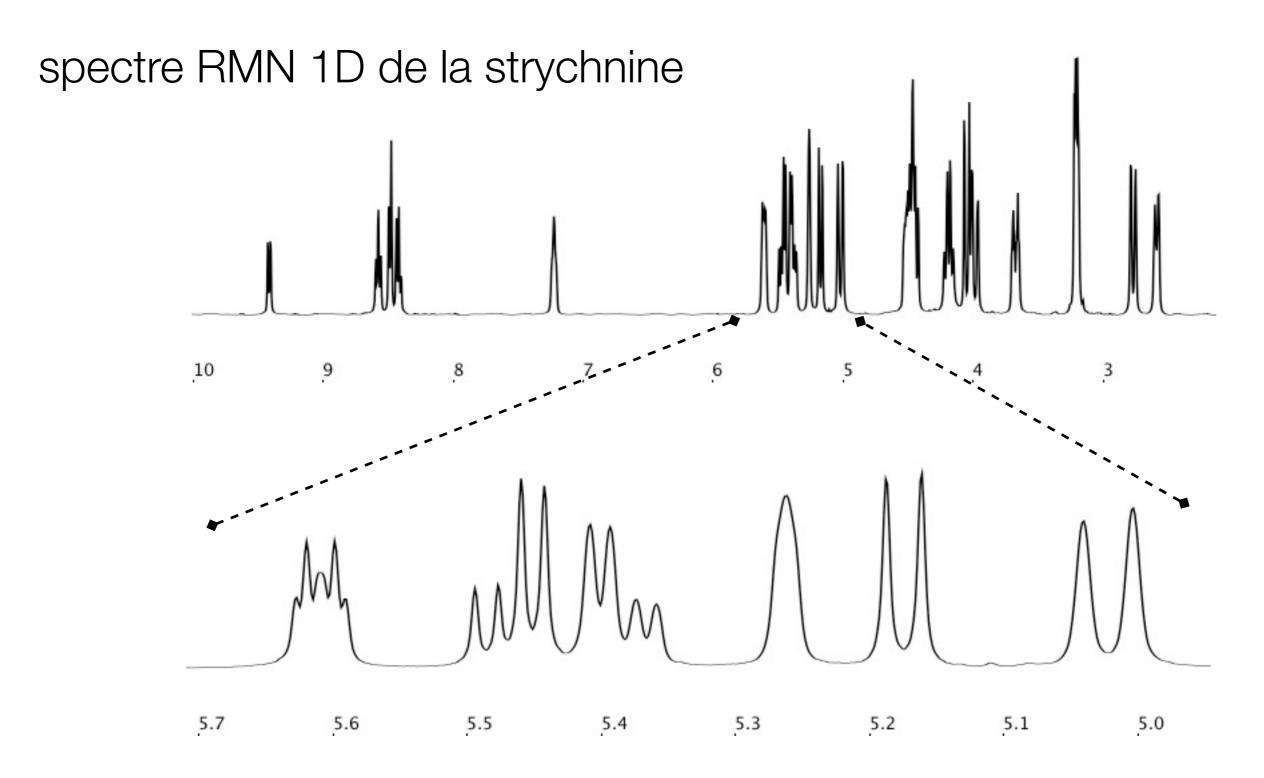
il existe différentes transformées

transformée de Fourier

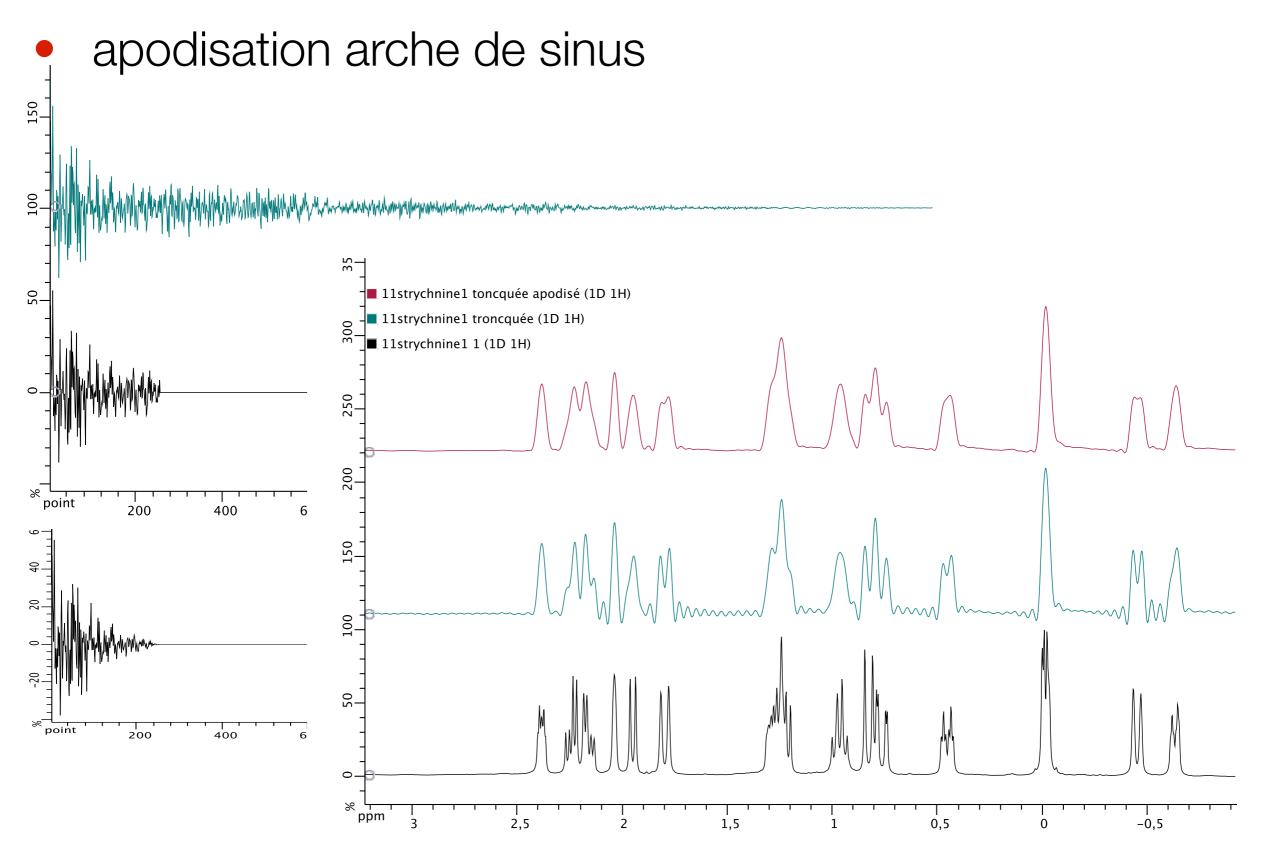
- inversible
- transformée d'Hadamard
 - sorte de transformée de Fourier sur {-1, 1}
 - inversible
- transformée de Hilbert
 - la transformée qui transforme un signal réel en la partie imaginaire du signal analytique
 - calculée via la transformée de Fourier
 - inversible
- transformée de Laplace
 - transformée sur les fonctions exponentielles
 - non-inversible
- transformée de Radon
 - transformée sur les fonctions projection
 - non-inversible



quelques exemples...



Zero-filling et apodisation



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Petite considération sur le zero-filling

comment ne pas perdre d'information

- 1 valeur == 1 élément d'information
- ne pas faire

FID : N points réelsFT(FID) : N/2 points complexesou N points complexesmodule : N/2 points réels

• faire

FID : N points réelsZero-Filling + N zéroFT(FID) : 2N points complexesmodule : N points réels

N valeurs indépendantes N valeurs indépendantes 2N valeurs non-indépendantes N/2 valeurs indépendantes

N valeurs indépendantes N non-nulles 2N valeurs non-indépendantes N valeurs indépendantes

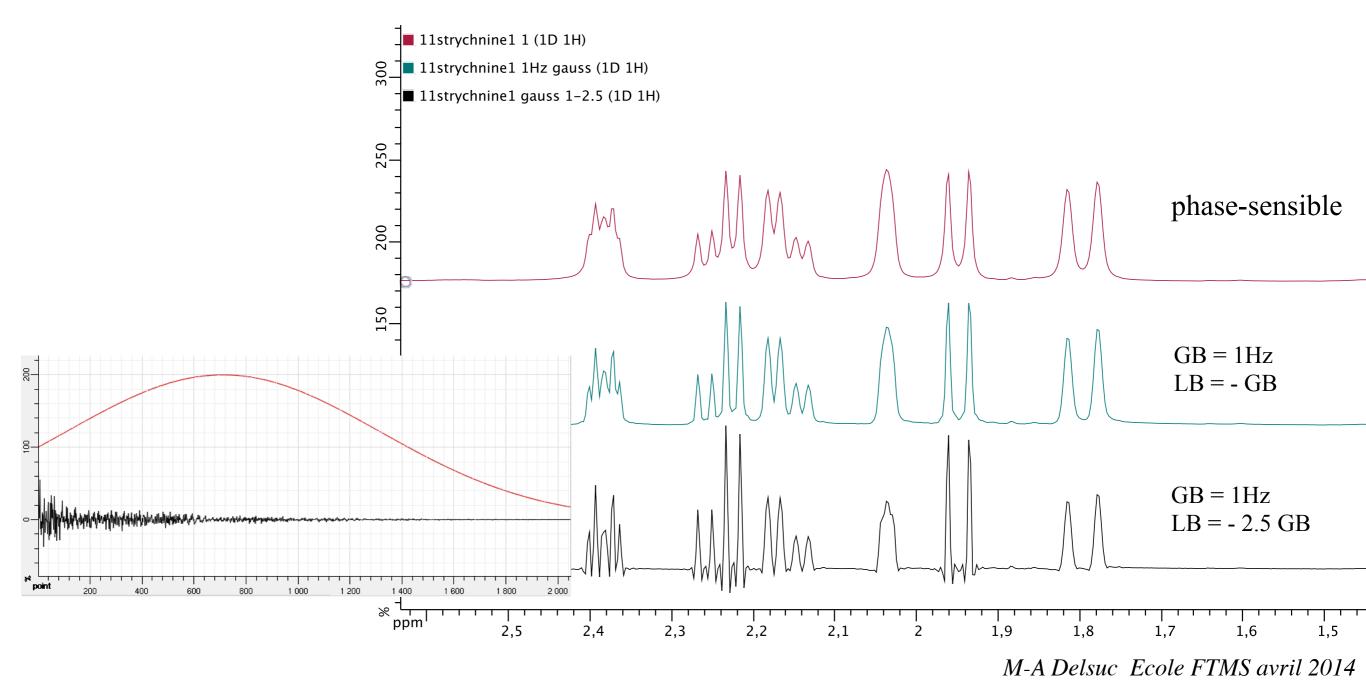
> rule of thumb : always zero-fill at least once

autres apodisations

• en mode phase sensible

 $e^{\frac{t}{\mathrm{LB}}}e^{\frac{-t^2}{\mathrm{GB}}}$

- apodisation "gaussienne"
- déconvolution par une lorentzienne reconvolution par une gaussienne



N<P la régularisation

Moins de points dans les données (N) que dans l'analyse (P) ???

- En fait très courant
 - en FT : «zerofilling» : faire la transformée en rajoutant des zeros
 - en image : interpolation nouvelles télés !
- \bullet On reconstruit l'image \tilde{s} la plus «jolie» possible
- Il faut un critère de beauté
 - information *a-priori* sur la mesure / sur les données

Position du problème

• modèle de mesure

 $y^{mes} = T_f(s) + \epsilon$

- fonction cible à minimiser
 - insuffisant pour trouver une solution unique
- critère supplémentaire
 - fonction de régulation à optimiser : R(s)
- Optimisation
 - miniser χ^2 sous la contrainte de R(s) minimum

 $\chi^2 = \sum \left(\frac{T_f(s) - y^{mes}}{\sigma}\right)^2$

$$\chi^2(s) + \lambda R(s)$$

Plusieurs possibilités:

• minimiser l'énergie E

$$E = \sum (s_i^2)$$

régularisation de Tikhonov T

(Γ opération linéaire) : courbure - smoothness - ...

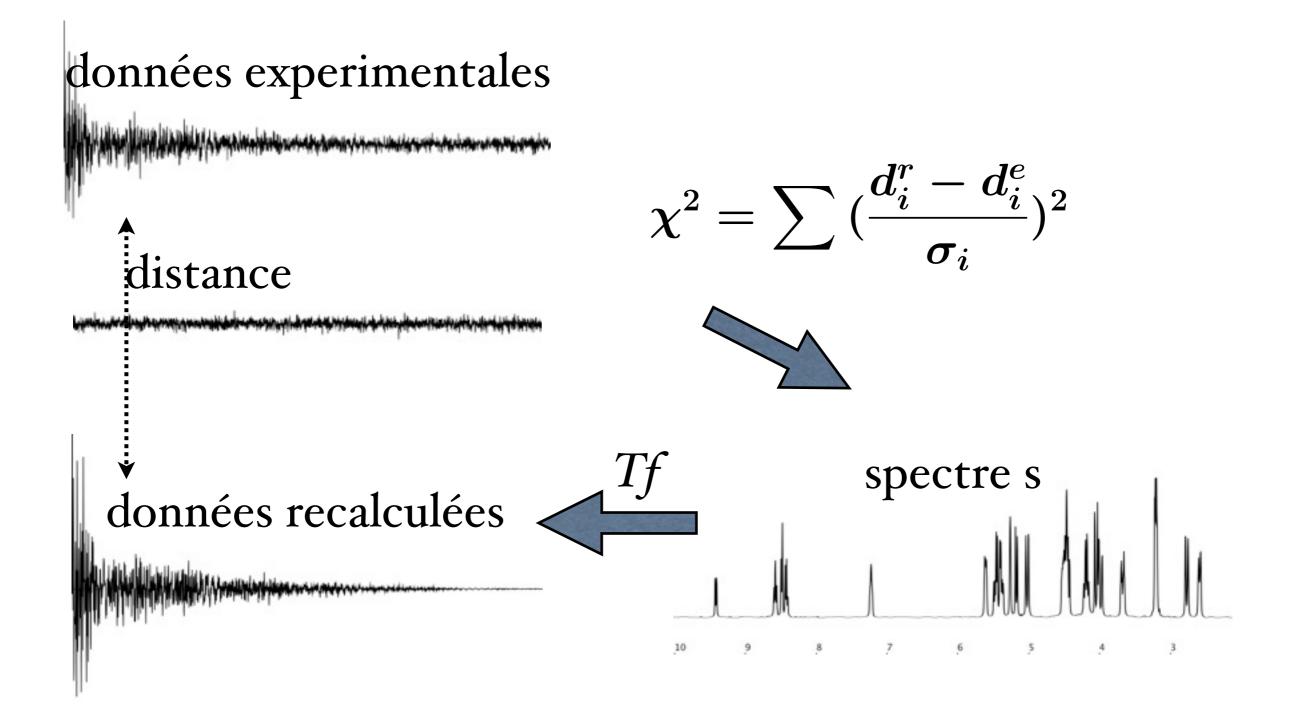
 $T = \sum (\Gamma(s)_i^2)$

- maximiser l'entropie
 - le spectre «le plus probable»
 - le spectre qui a le moins d'«information» (sens de Shanon)

- maximiser la simplicité
 - la parcimonie : le nombre de signaux au dessus du bruit.

$$S = -\sum(s_i \log(s_i))$$

principe de l'approche inverse

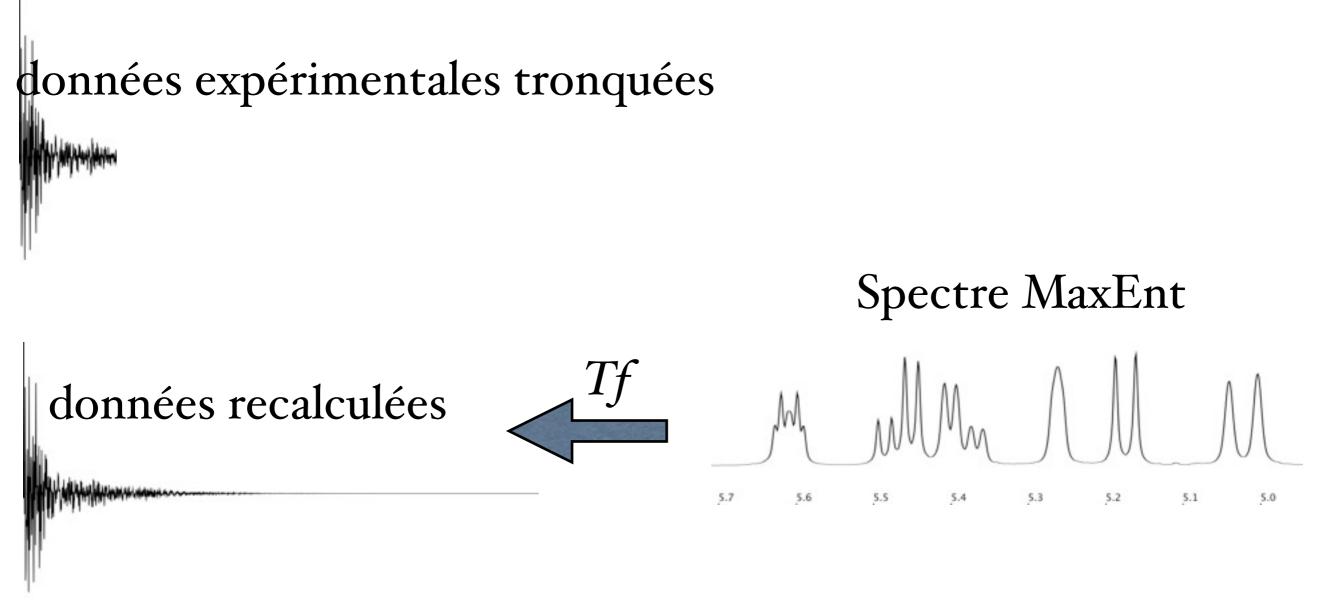


Signal Entropy

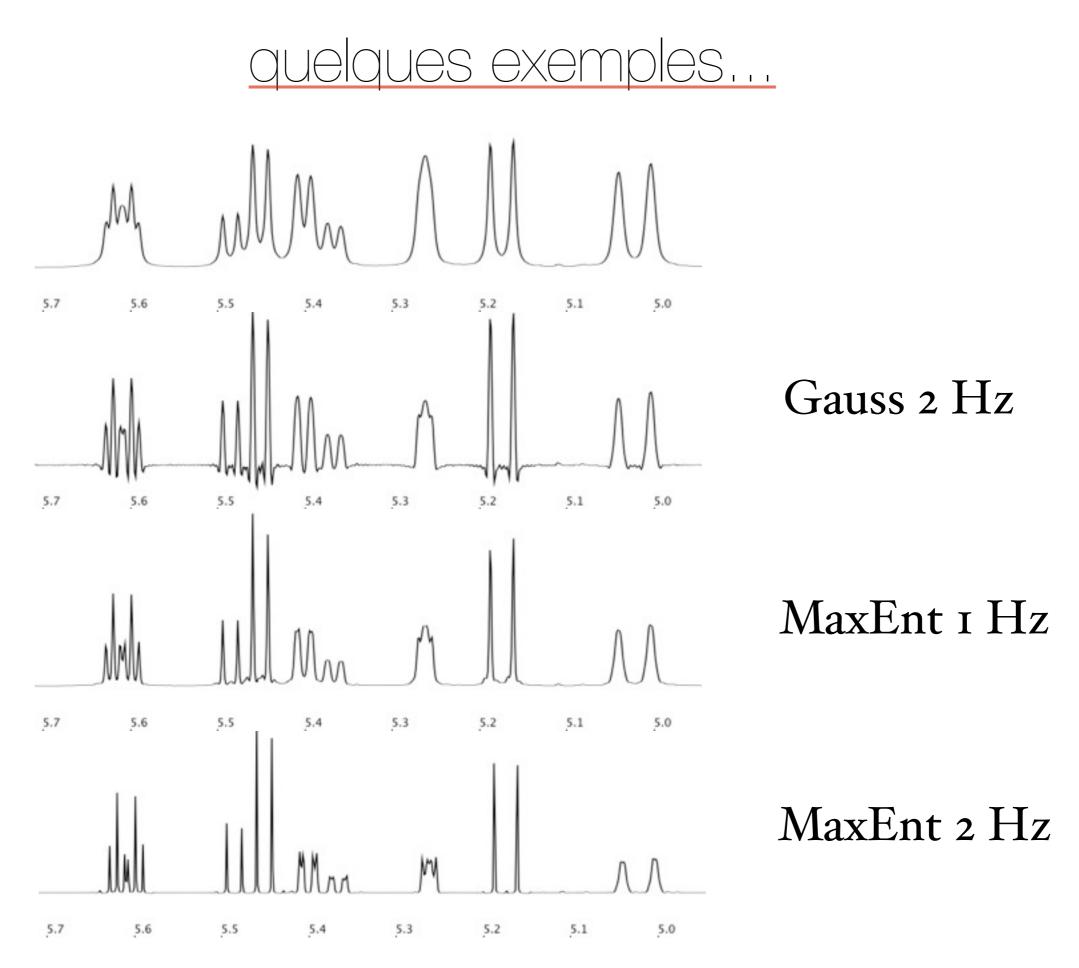
- Among all the spectra which adapt the data down to the noise, I choose :
 - the one with the minimum information (Shannon sense)
 - the most probable
 - => the one which maximize the signal entropy

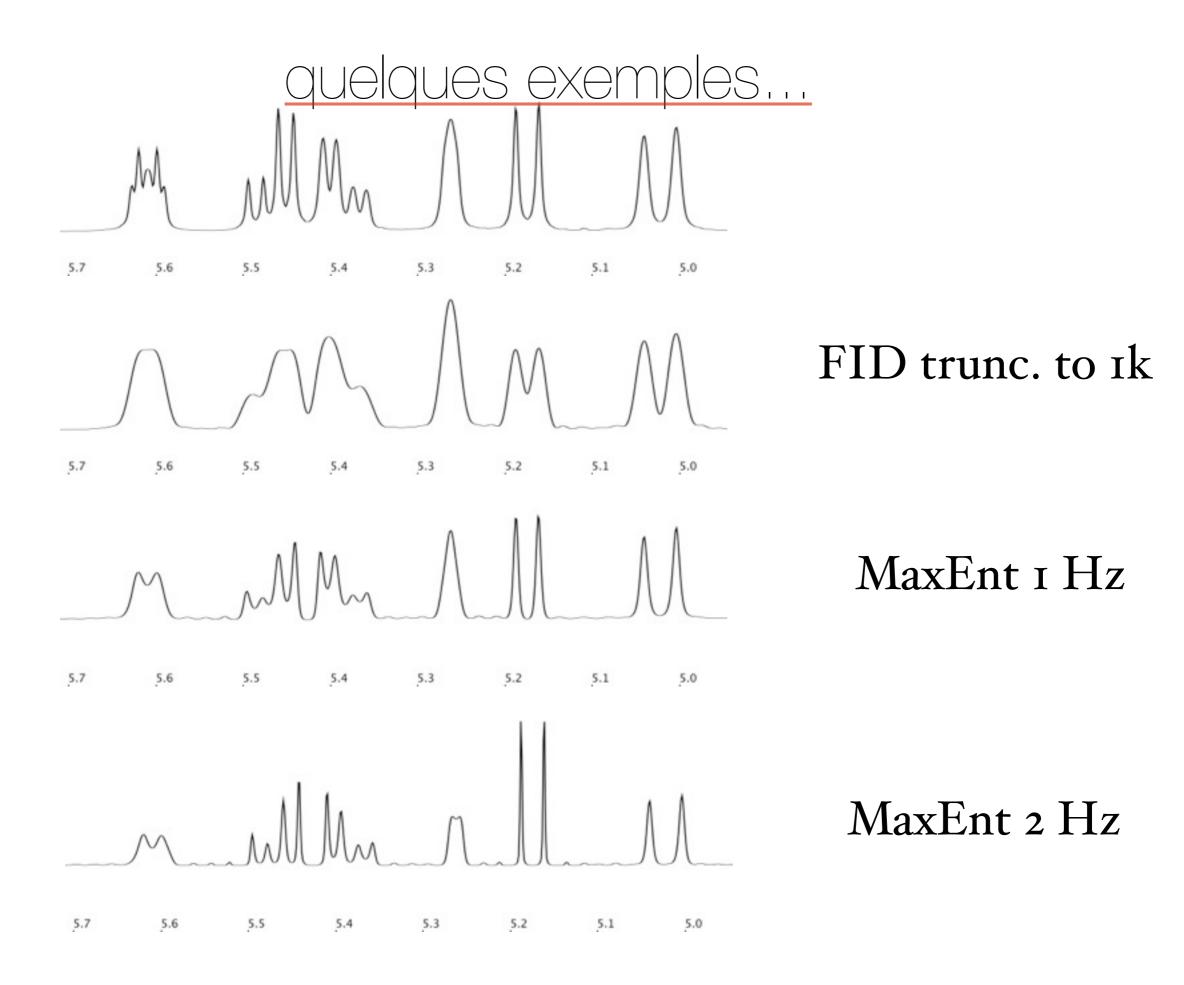
$$S = -\sum p_i \log(p_i)$$
 with $p_i = rac{f_i}{\sum f_i}$

extension par approche inverse



 χ^2 calculé sur les données mesurées



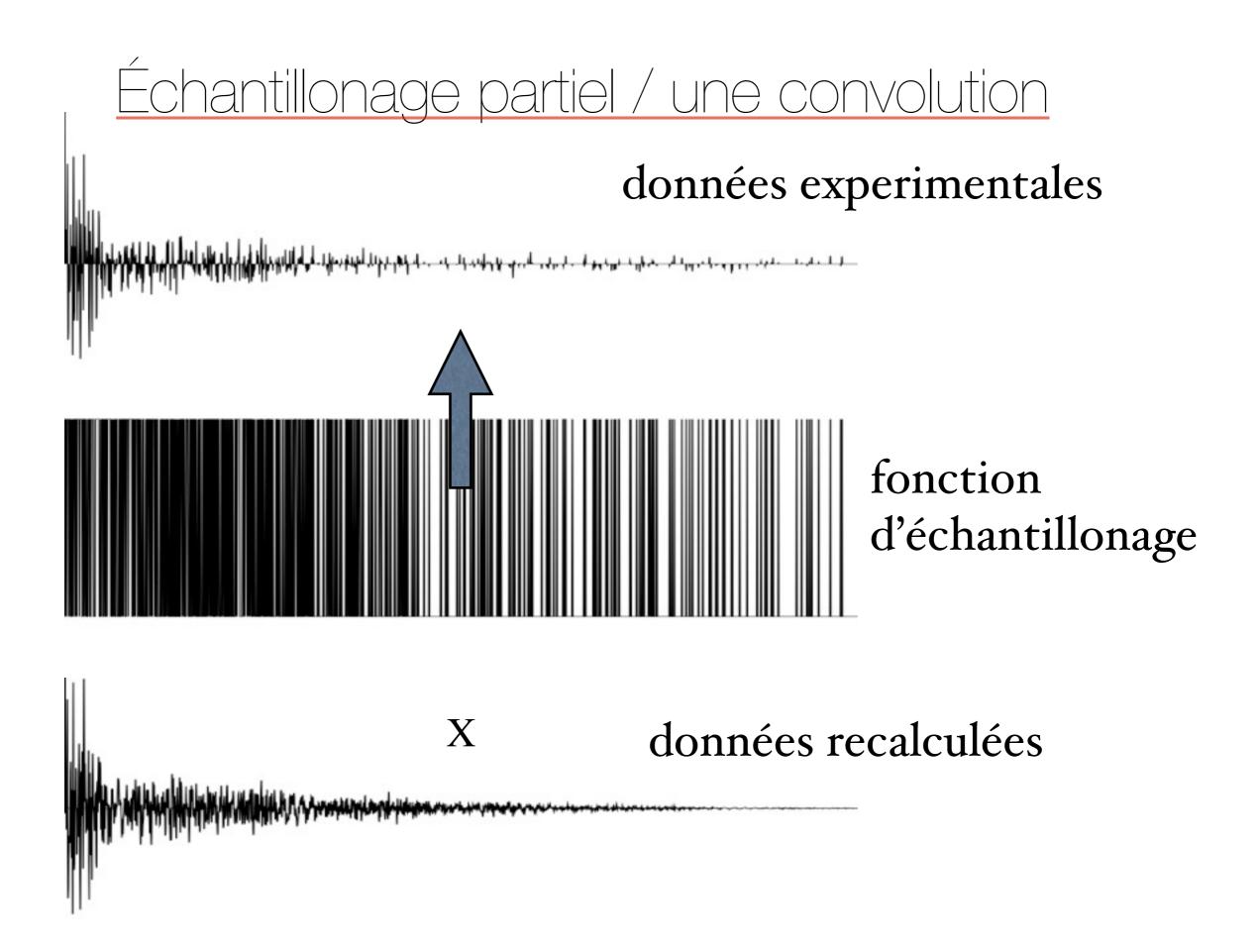


Échantillonage partiel vu comme une convolution

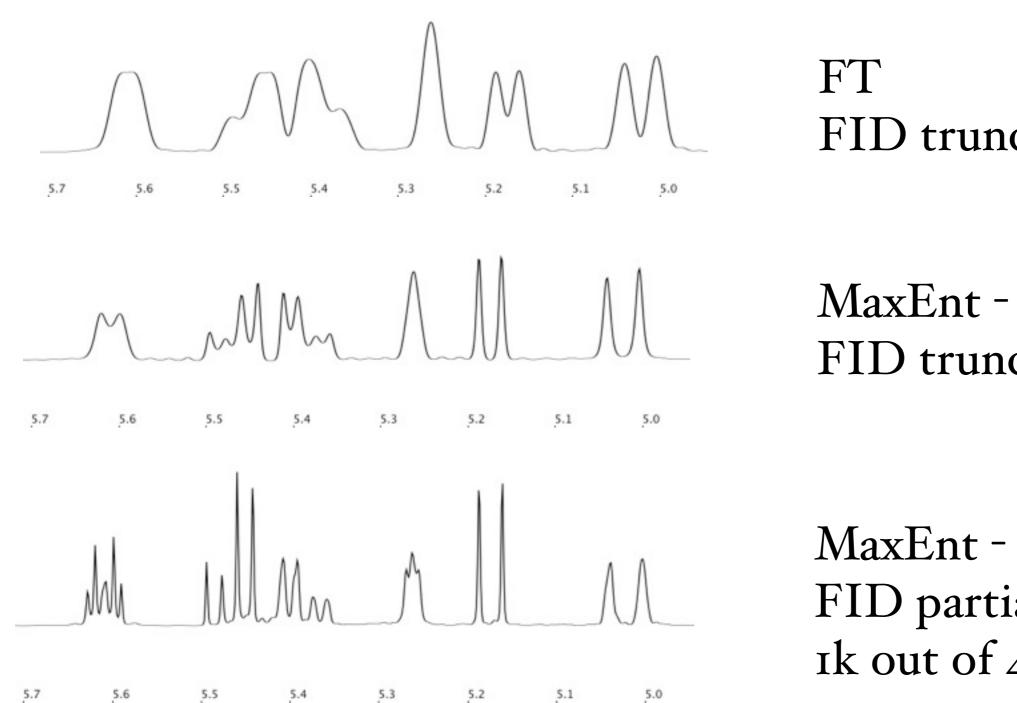
données classiques



fonction d'échantillonage



strychnine encore ...



FID trunc. to ik

MaxEnt - IHz FID trunc. to ik

MaxEnt - 1Hz FID partially sampled ık out of 4k

An automated tool for maximum entropy reconstruction of biomolecular NMR spectra

NATURE METHODS | VOL.4 NO.6 | JUNE 2007 | 467

Mehdi Mobli^{1–3}, Mark W Maciejewski^{1,3}, Michael R Gryk¹ & Jeffrey C Hoch¹

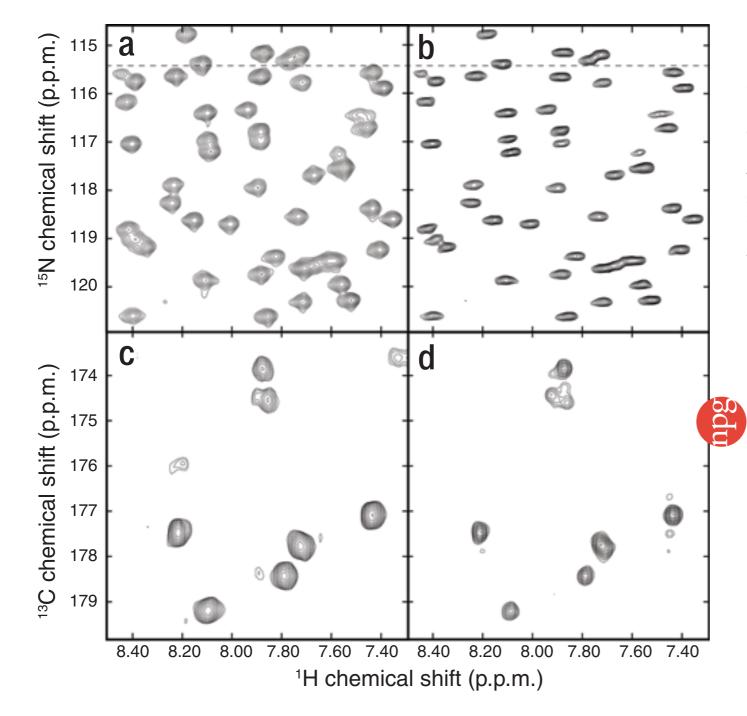


Figure 1 | Automated MaxEnt reconstruction can dramatically reduce data collection time or improve resolution. $(\mathbf{a}-\mathbf{d})^{15}N$ heteronuclear single quantum correlation (HSQC; \mathbf{a}, \mathbf{b}) and HNCO (\mathbf{c}, \mathbf{d}) spectra are shown for DNA polymerase X obtained using conventional processing (linear-prediction extrapolation and sinebell apodization; \mathbf{a}, \mathbf{c}), and automated MaxEnt reconstruction using linewidth deconvolution to improve resolution without the sensitivity losses characteristic of apodization (\mathbf{b}), and nonuniform sampling to achieve a sevenfold reduction in data acquisition time (\mathbf{d}). Two-dimensional cross-sections of the three-dimensional spectrum in \mathbf{c} and \mathbf{d} correspond to the ¹⁵N frequency indicated by the dashed lines in \mathbf{a} and \mathbf{b} .



- Maximiser la "parcimonie" du signal
 - la simplicité
 - minimiser la somme de *abs(x_i)*
 - K : nombre de signaux non nul

Nécessite certaine conditions

- sur la fonction de transfer inversible RIP
- Alors la reconstruction peut-être exacte (si pas de bruit)
 - papier E.Candès et T.Tao 2006
 - example
 - SL0



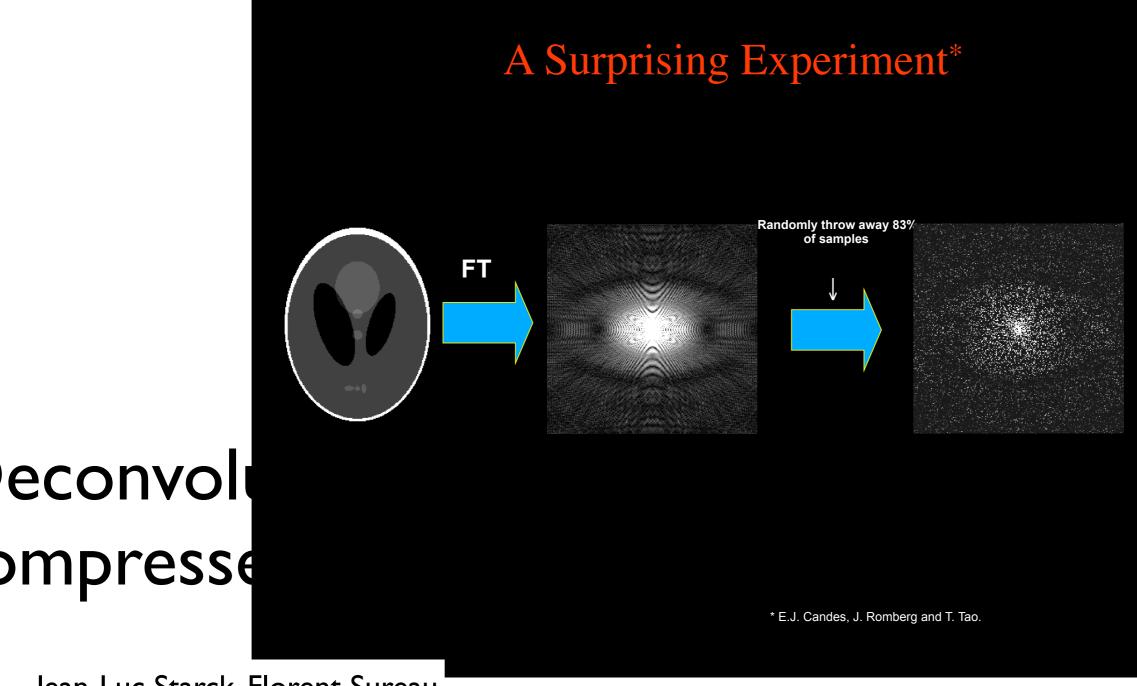
E.Candès T.Tao 2006

Robust Uncertainty Principles: Exact Signal Reconstruction from Highly Incomplete Frequency Information

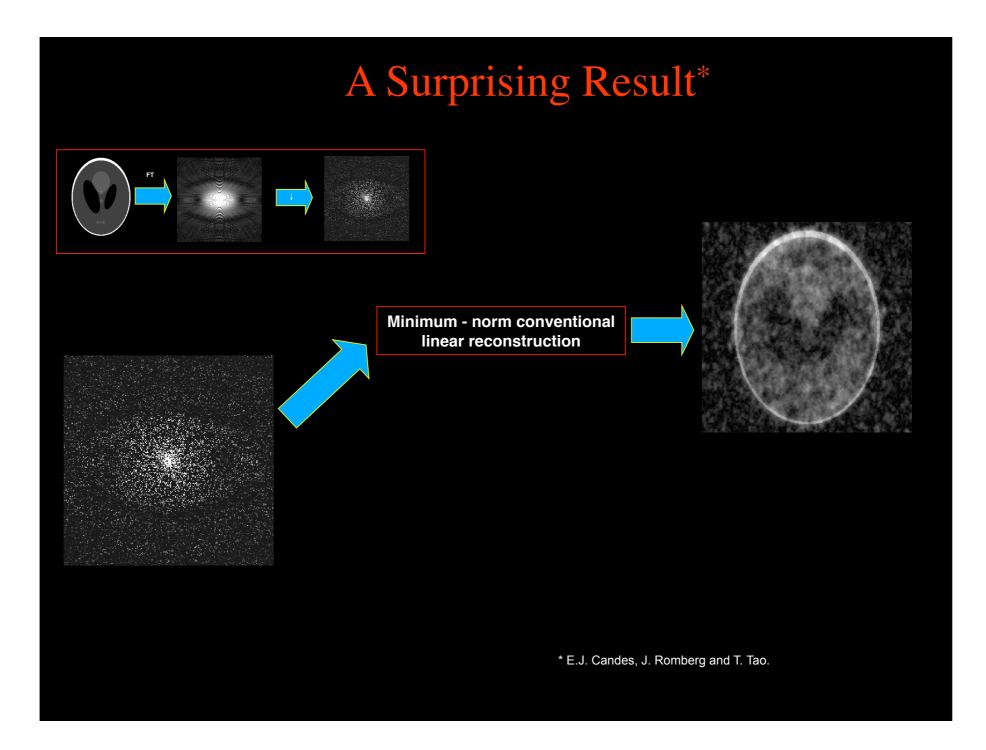
Emmanuel Candes[†], Justin Romberg[†], and Terence Tao^{\sharp}

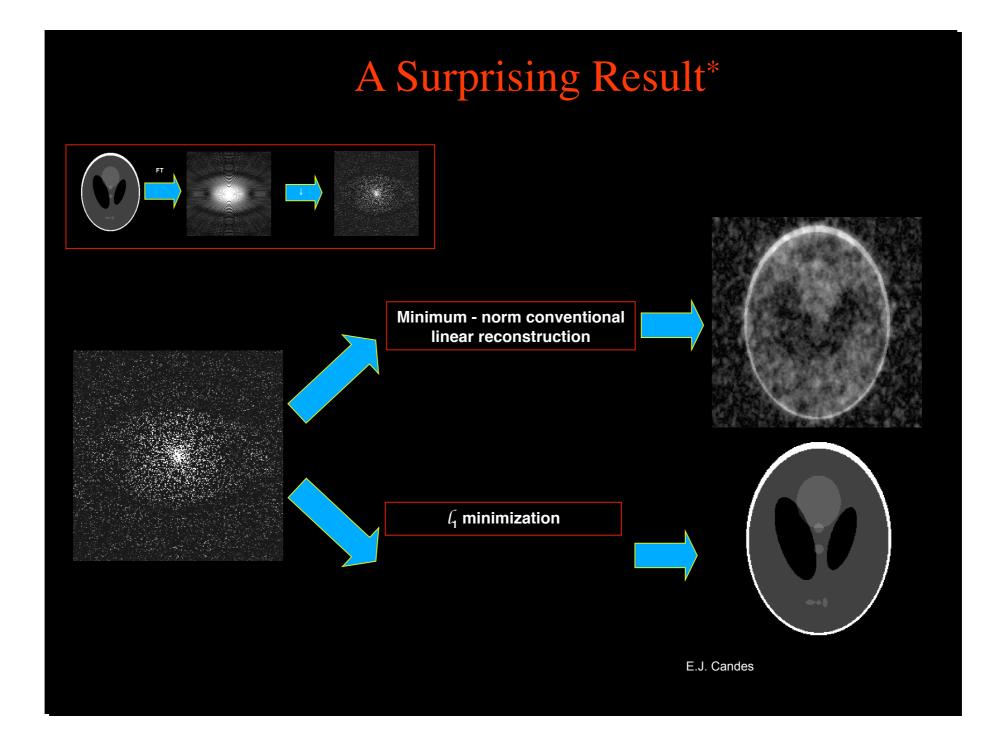
† Applied and Computational Mathematics, Caltech, Pasadena, CA 91125
‡ Department of Mathematics, University of California, Los Angeles, CA 90095

E. J. CANDÈS, J. ROMBERG, AND T. TAO, Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information, IEEE Trans. on Information Theory, 52 (2006), pp. 489–509.



Jean-Luc Starck, Florent Sureau J. Bobin, N. B:





Compressed Sensing

régulation : données simples/creuses - sparsity -

- K signaux N mesures P points dans le spectres
- hypothèse de peu de signaux : K<<N<<P

• Le problème devient une simple optimisation convexe $\min(\|s\|_1)$ avec $\|y - Ts\|_2 < \epsilon$

• norme
$$l_l$$
 ou l_o mais pas l_2
 $\|s\|_1 = \sum |s_i| \qquad \|s\|_p = \left(\sum s^p\right)^{\frac{1}{p}}$

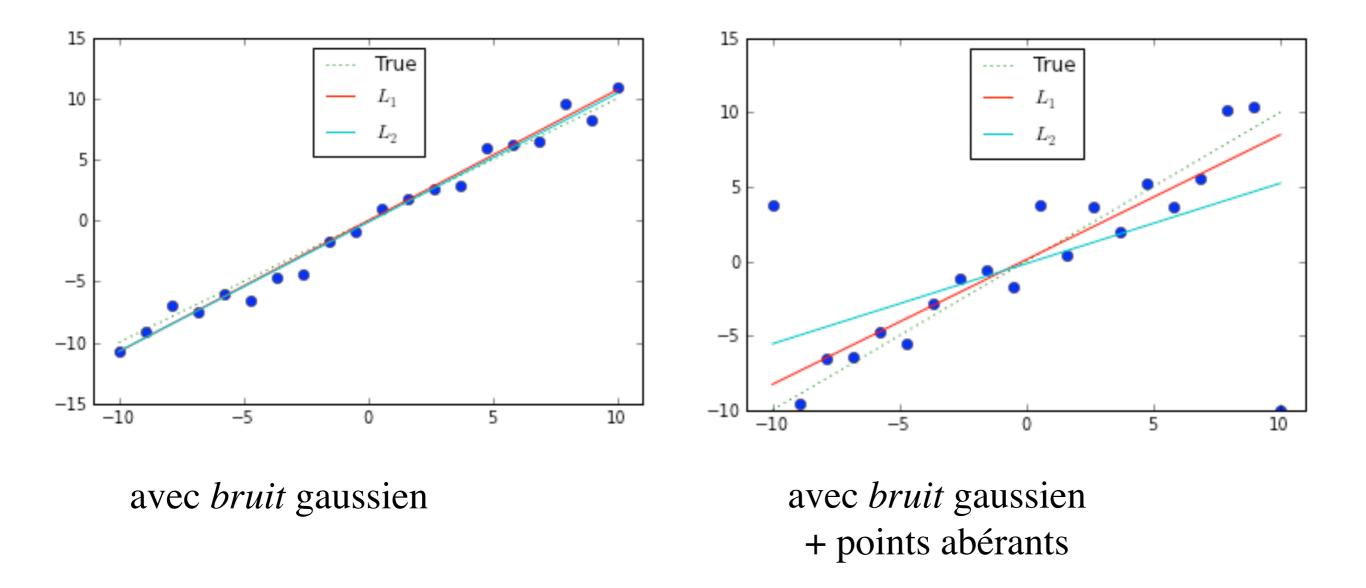
Dans certaines conditions

R.I.P. Restricted Isometry Property

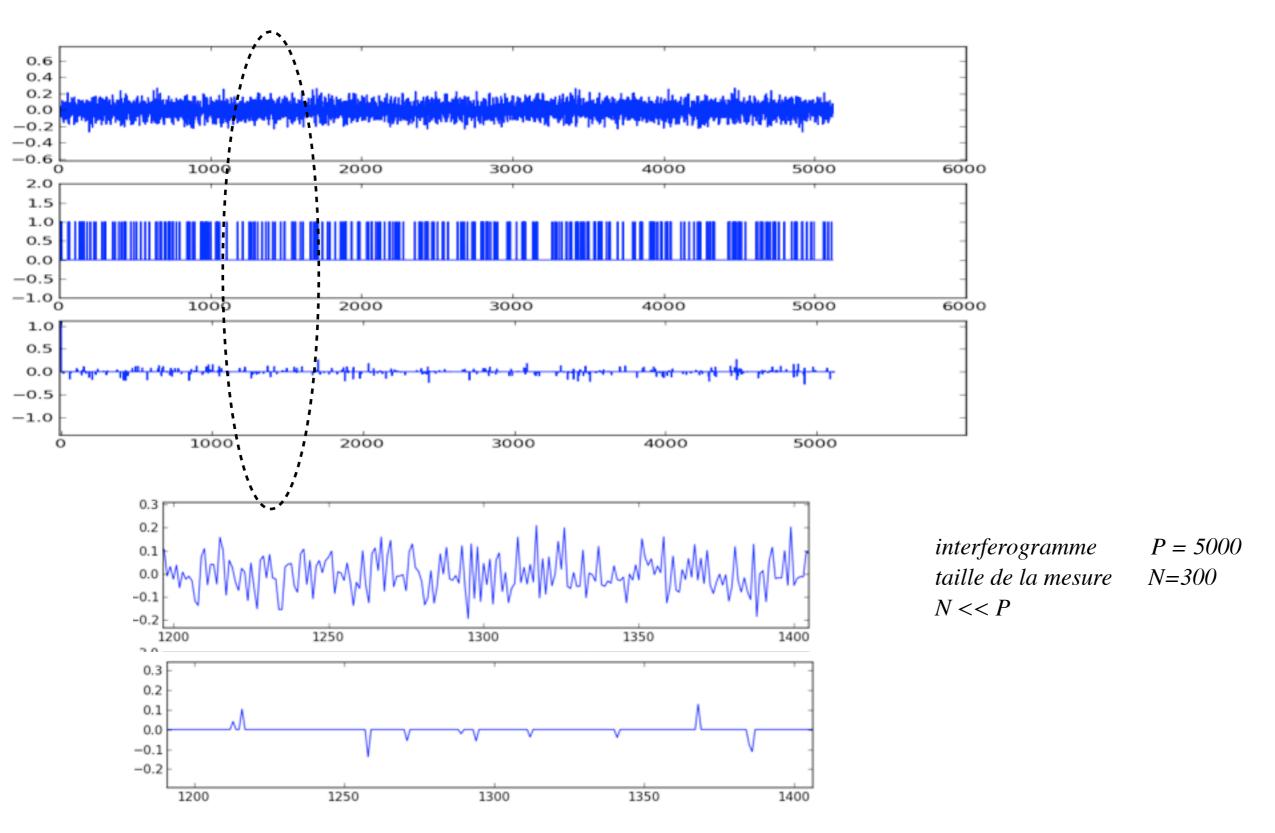
- dispersion (FT par exemple)
- pseudo-inversible
- linéaire

En exemple LIVE

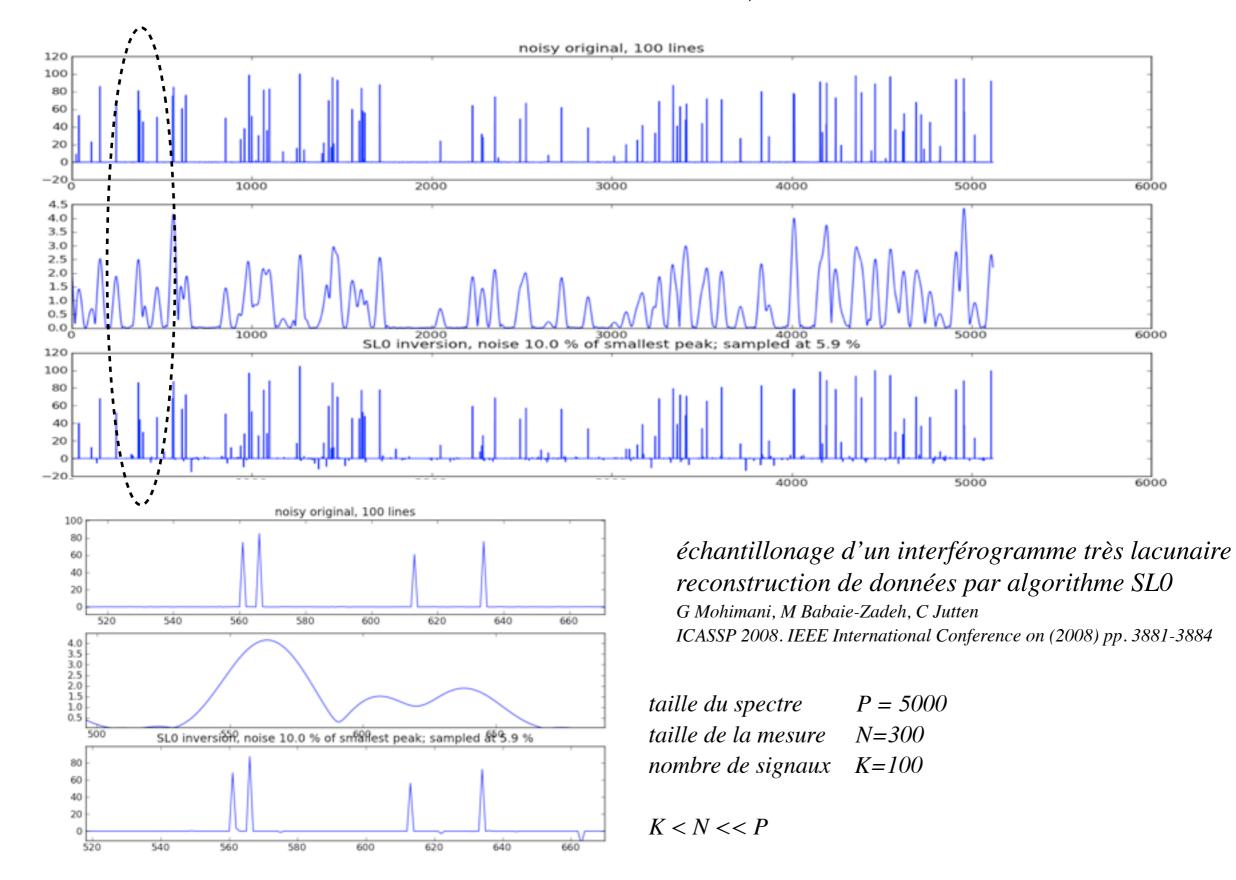
Comparaison minimisation L_1 vs L_2



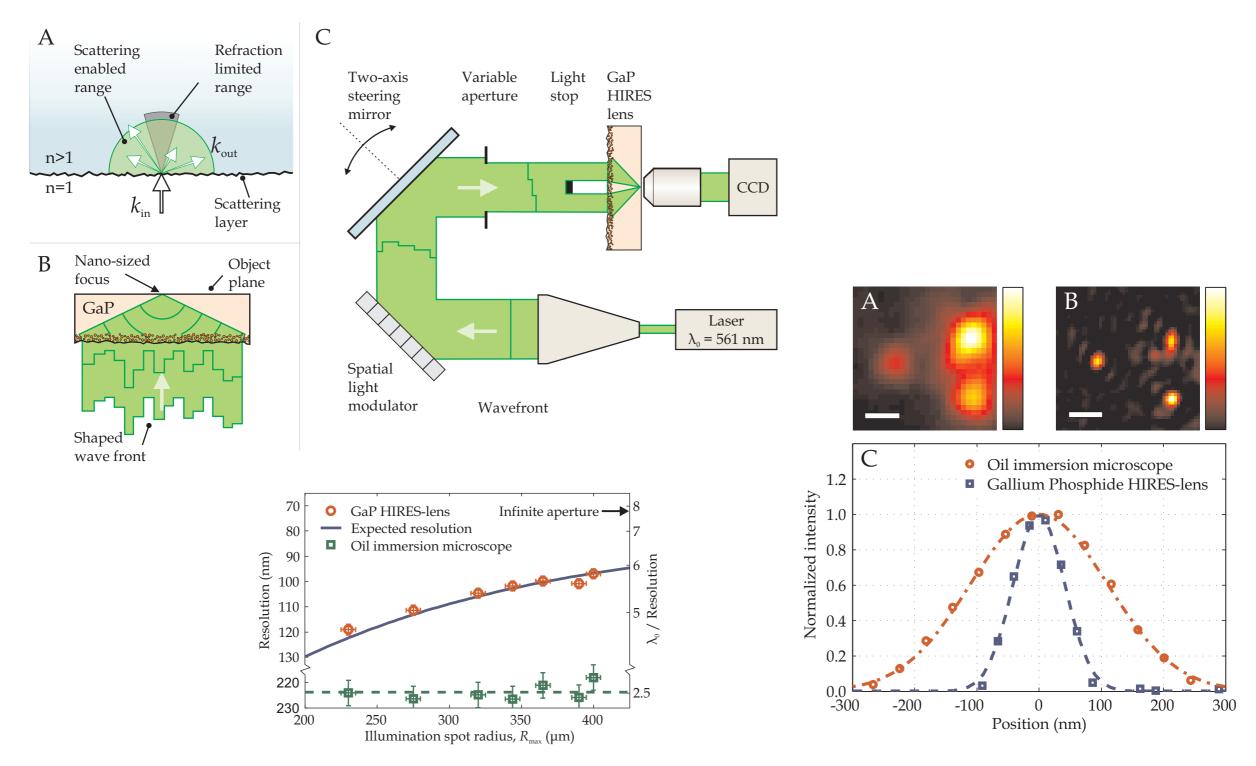
échantillonnage extrême



<u>Un exemple</u>

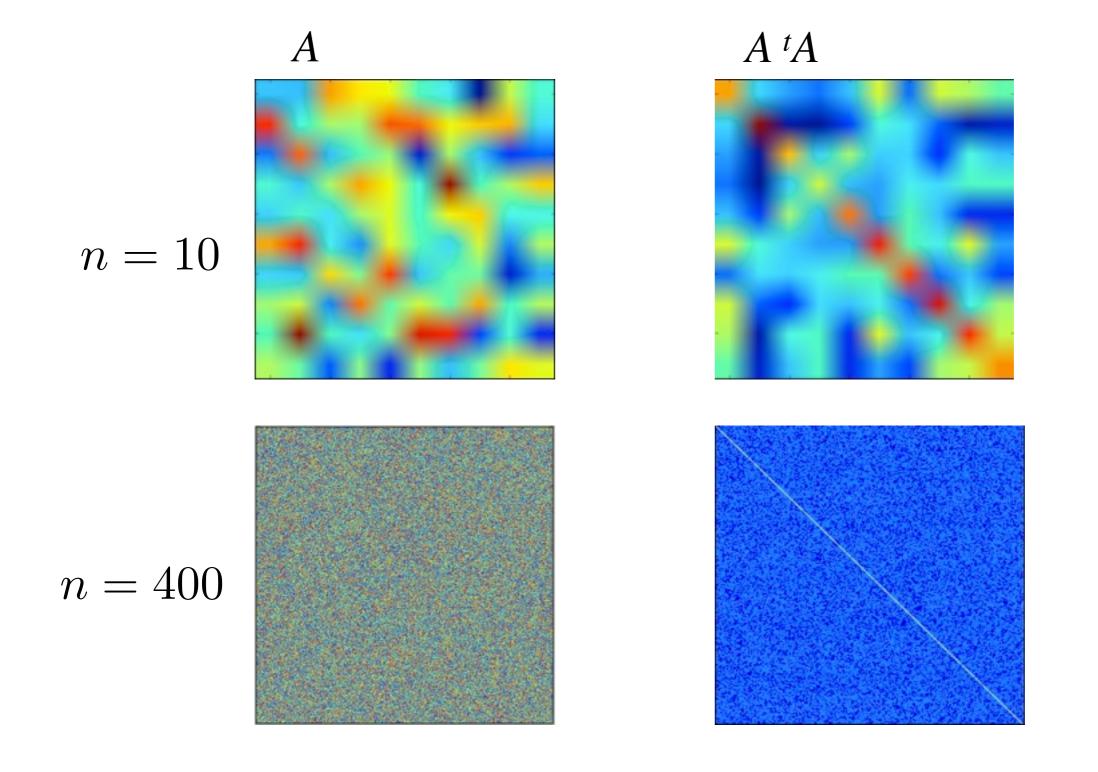


C'est assez fort !

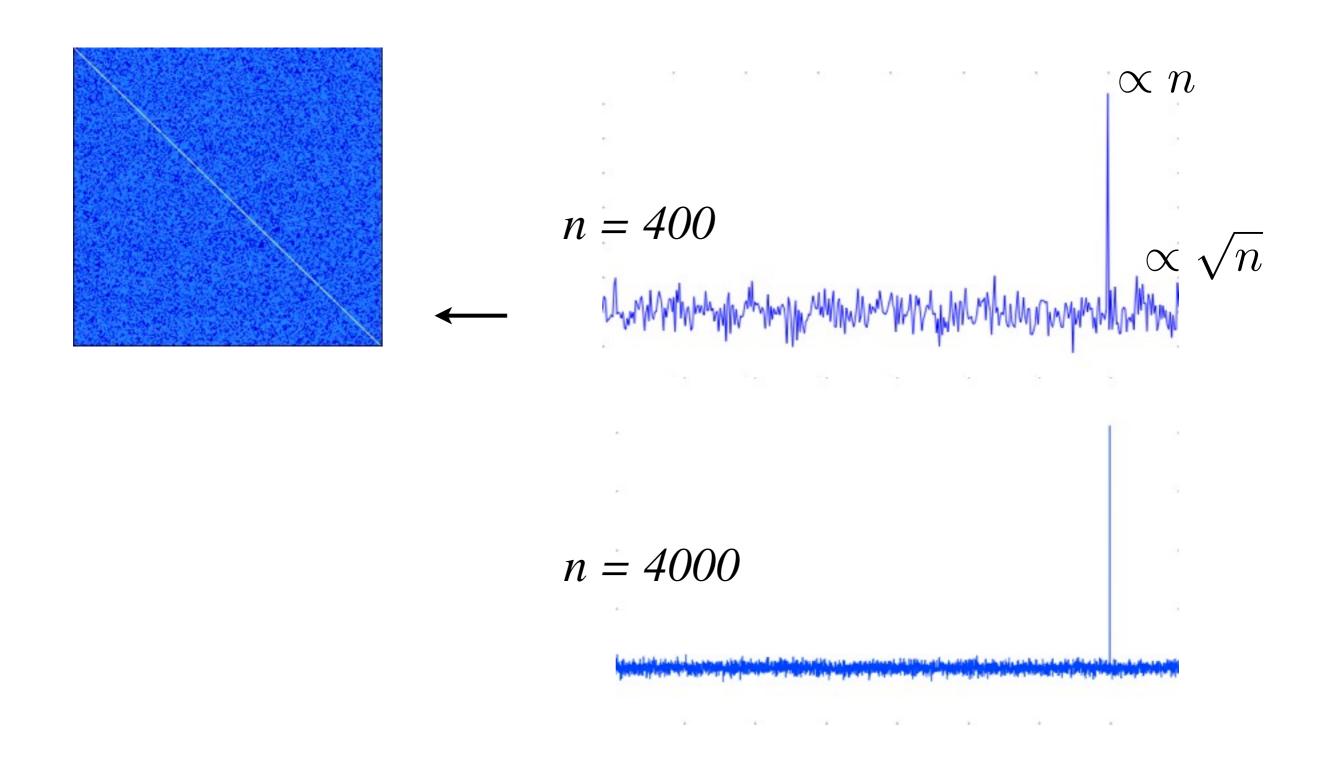


Putten et al. Scattering Lens Resolves sub-100 nm Structures with Visible Light. arXiv (2011) vol. physics.optics

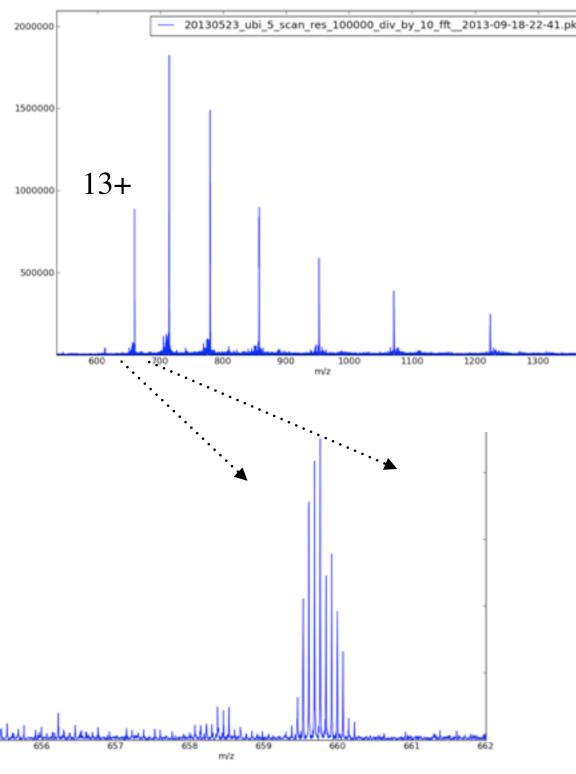
matrices aléatoires



grandes matrices aléatoires

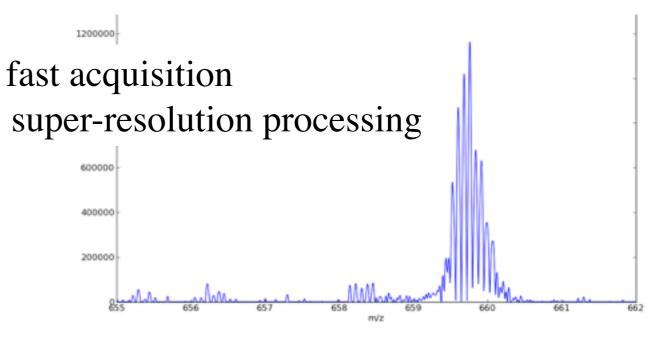


Super-resolution / fast acquisition



HiRes acquisition standard processing Orbitrap Ubiquitin spectrum (col. J. Chamot-Rooke Institut Pasteur)

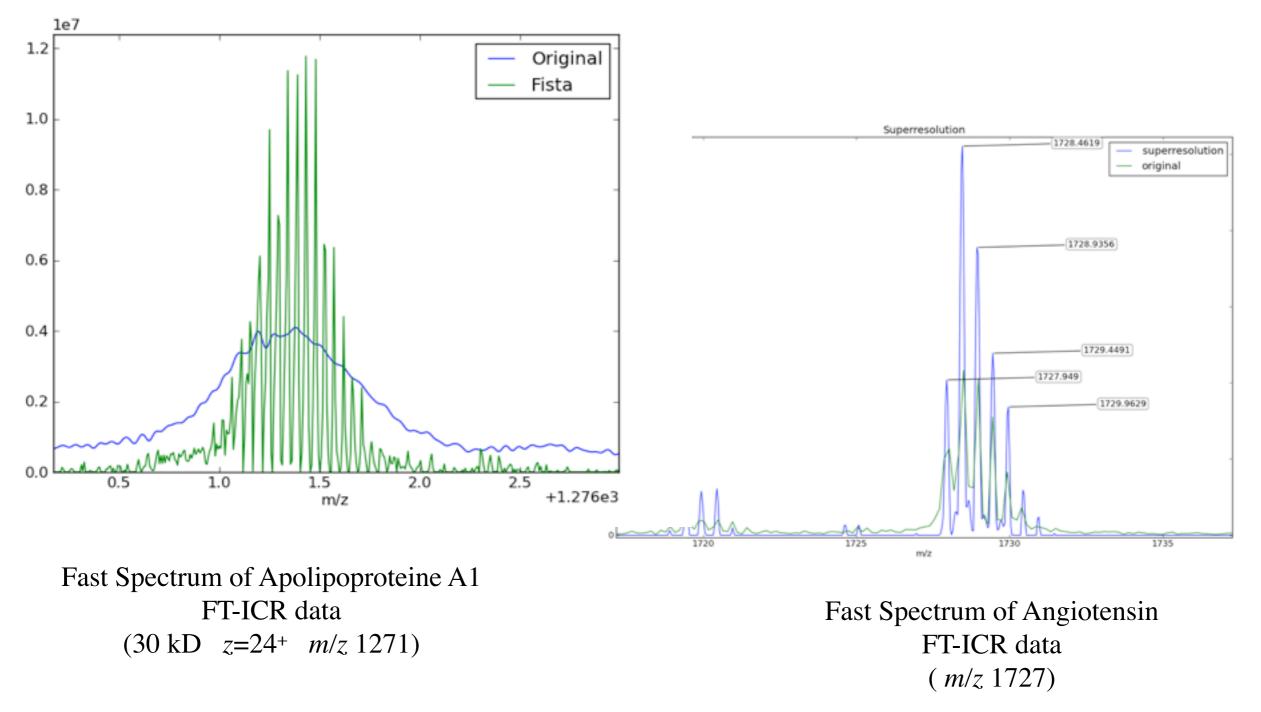
fast acquisition (10x faster) standard processing



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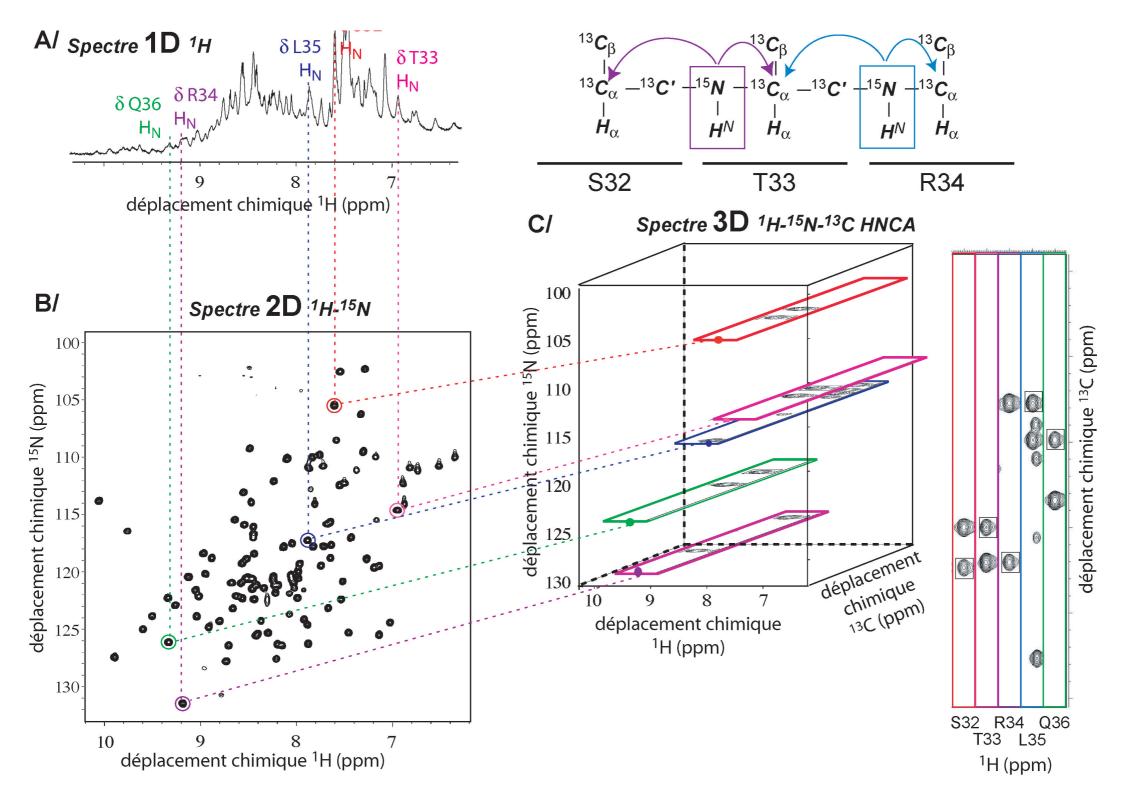
other example

Standard FT vs Sur-resolution processing



(col. C. Rolando Univ Lille)

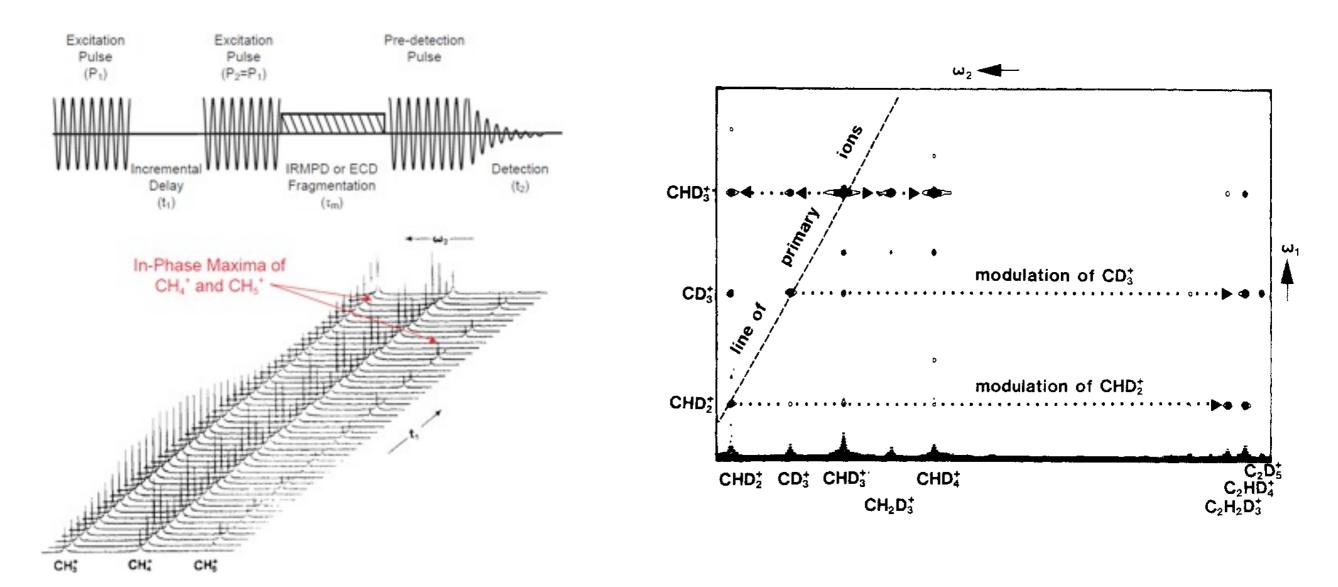


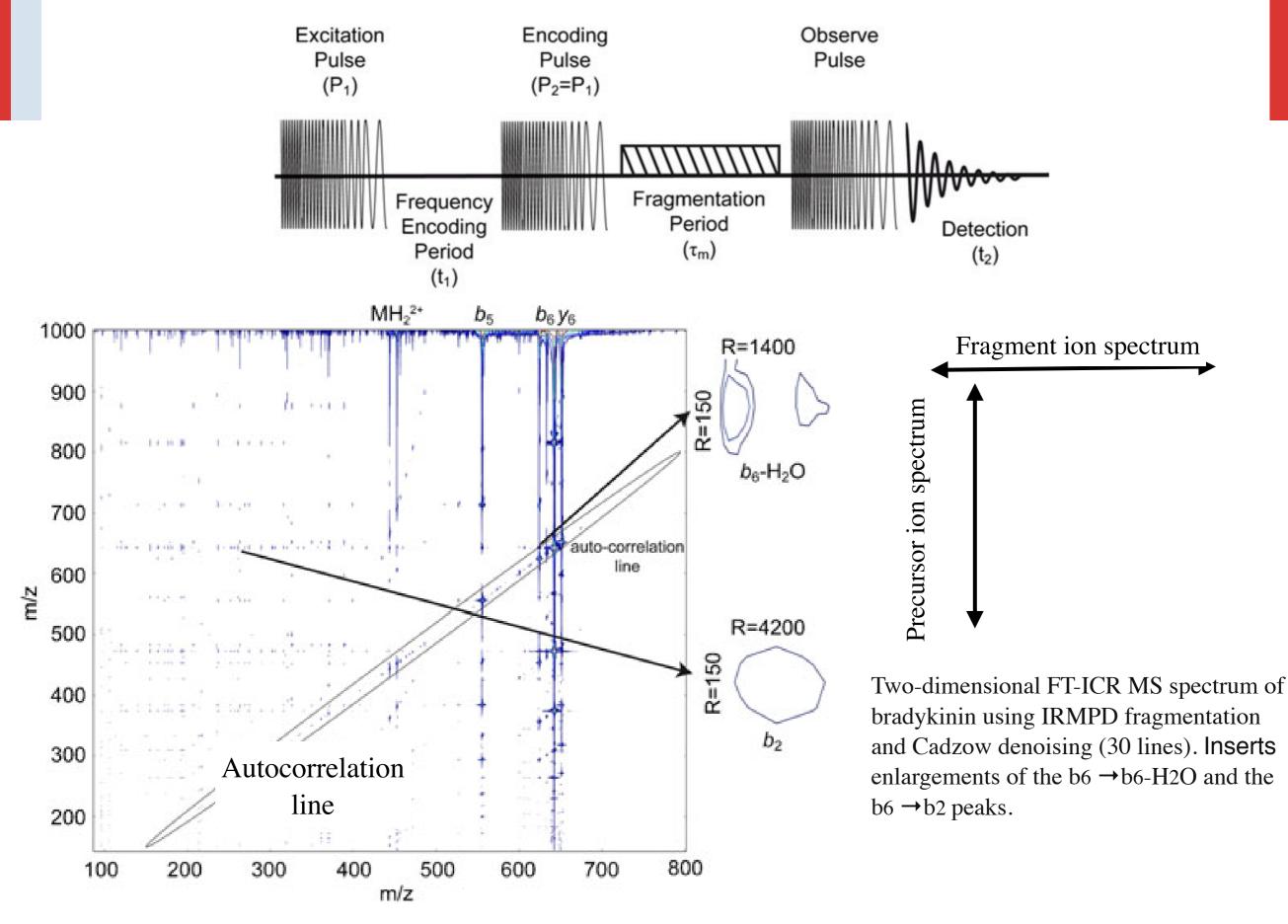




principle of 2D FTICR proposed in 1987-88 nearly as old as 2D NMR

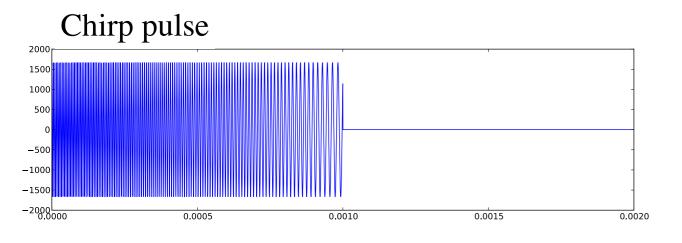
P Pfändler and G Bodenhausen and J Rapin and R Houriet and T Gäumann *Chem Phys Let* (1987) vol. 138 (2) 195-200 P Pfaendler, G Bodenhausen, J Rapin, M Walser, T Gaümann J Am Chem Soc (1988) vol. 110 (17) 5625-5628





van Agthoven, M. A., Delsuc, M.-A., Bodenhausen, G. & Rolando, C. *Anal Bioanal Chem* **405**, 51–61 (2013).

FT-ICR simulator



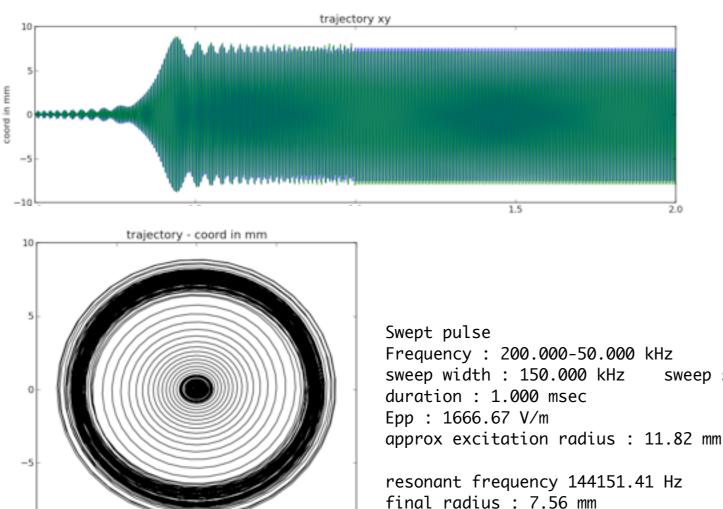
ion trajectory

-10-10

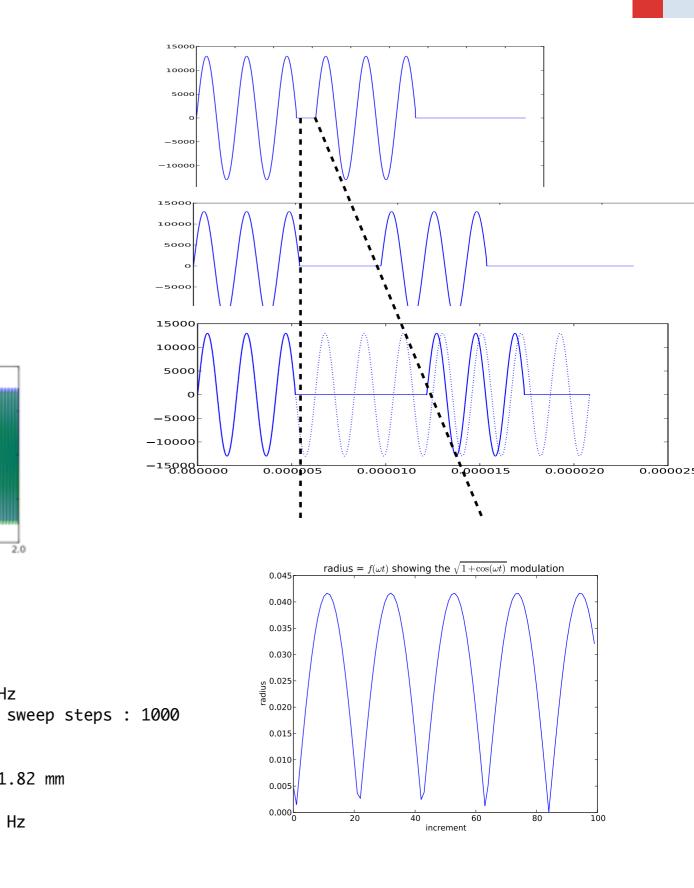
-5

0

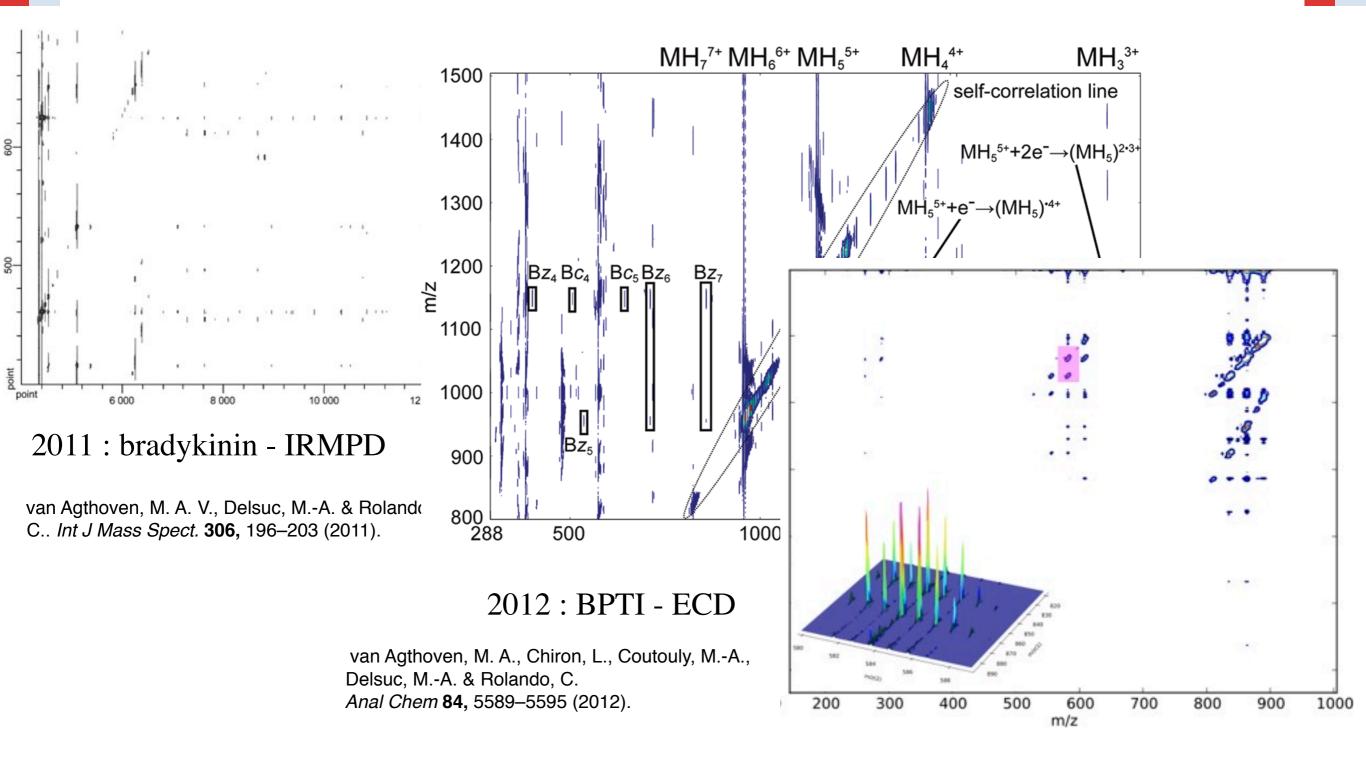
5



10



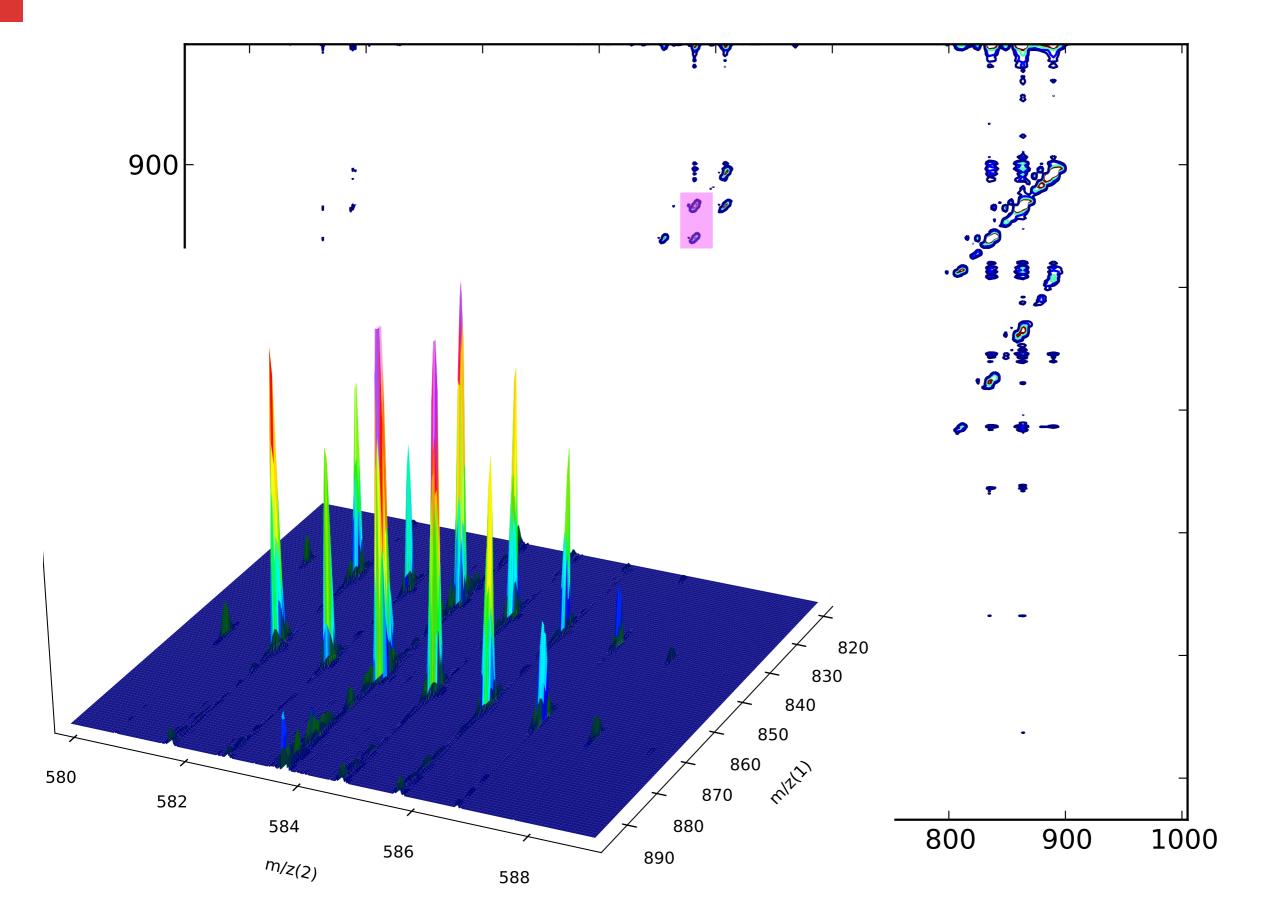
Improvements over time



2014 : triglycerid - high resolution

Chiron, L., van Agthoven, M. A., Kieffer, B., Rolando, C. & Delsuc, M.-A. *Proc Natl Acad Sci USA* **111**, 1385–1390 (2014).

Tri Acyl Glycerol (TAG)

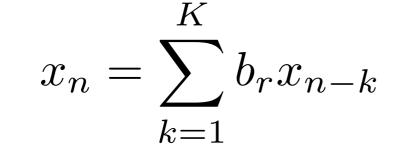


Le bruit

Classique

- additif blanc gaussien centré
- hétéroscédasticité
 - bruit multiplicatif
 - bruit de scintillation
 - bruit d'instrument
- traitement explicite dans les modèles mathématiques
- ce que je ne connais pas ce qui est en dehors de mon modèle
 - erreur
 - biais

Débruitage par la Procédure de Cadzow



$$\begin{bmatrix} x_1 & x_2 & \dots & x_P \\ x_2 & x_3 & \dots & x_{P+1} \\ x_3 & x_4 & \dots & x_{P+2} \\ x_4 & x_5 & \dots & \ddots \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ x_{L-P} & x_{L-P+1} & \dots & x_{L-1} \end{bmatrix} \begin{bmatrix} b_P \\ b_{P-1} \\ b_{P-2} \\ \vdots \\ b_1 \end{bmatrix} = \begin{bmatrix} x_{P+1} \\ x_P \\ x_{P-1} \\ \vdots \\ x_L \end{bmatrix}$$

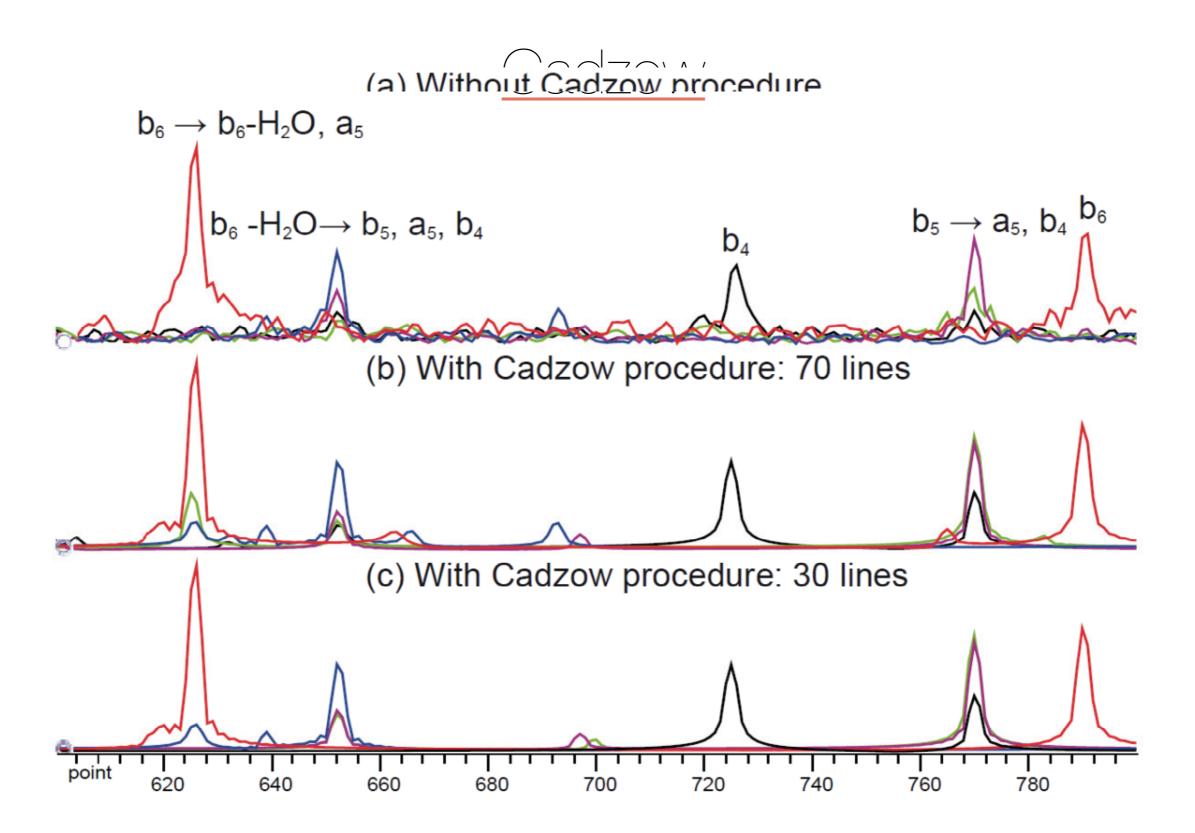
Equation de Prédiction Linéaire (LP)

- ⇔ somme de K sinusoïdes amorties
- expression matricielle du problème LP Hb = x
- H est décomposée en valeurs singulières

$$H = U \times \Sigma \times V^*$$

- puis tronquée à K valeurs propres $\tilde{H} = U \times \tilde{\Sigma} \times V^*$

équivalent à $\|\tilde{H}\|_o = K$



Vertical precursor ions spectra from the 2D IRMPD FT-ICR MS spectrum of bradykinin: *b*6 (red), *b*6-H2O (blue), *b*5 (pink), *a*5 (green), *b*4 (black) (a) without Cadzow procedure, (b) with Cadzow algorithm for 70 lines and (c) for 30 lines [32]

Improving the detection

Sensitivity of the measure is governed by Signal/Noise ratio

- \Rightarrow increase signal
- \Rightarrow reduce noise

Noise sources

- "standard"
 - coming from the electronic on the apparatus
 - \Rightarrow acquire more scan = takes time
- scintillation noise
 - comes from the sample
 - \Rightarrow no counter action during acquisition
 - preponderant in 2D (t1-noise in NMR)

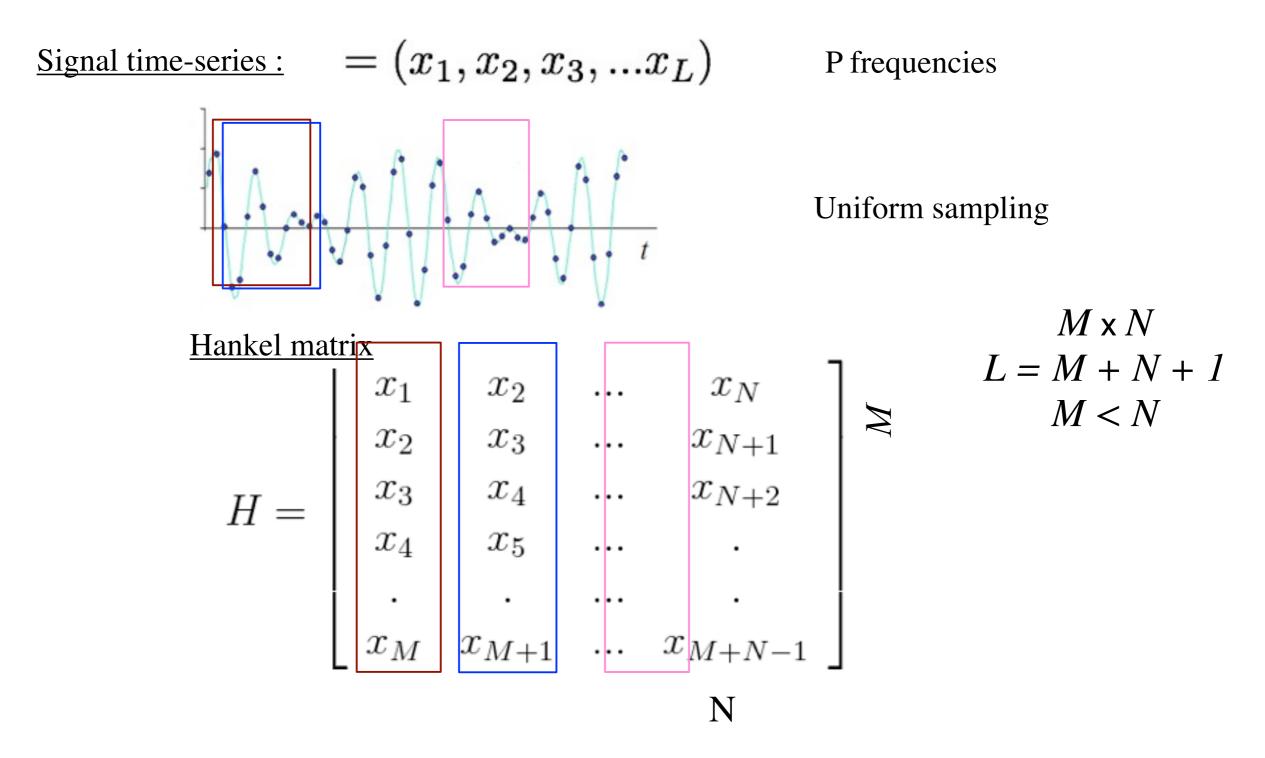
Impact

- better detection of weaker compounds
- better coverage in bottom-up proteomics
- better detection of PTM
- faster acquisition

 $S/N = \sqrt{N_{scan}}$

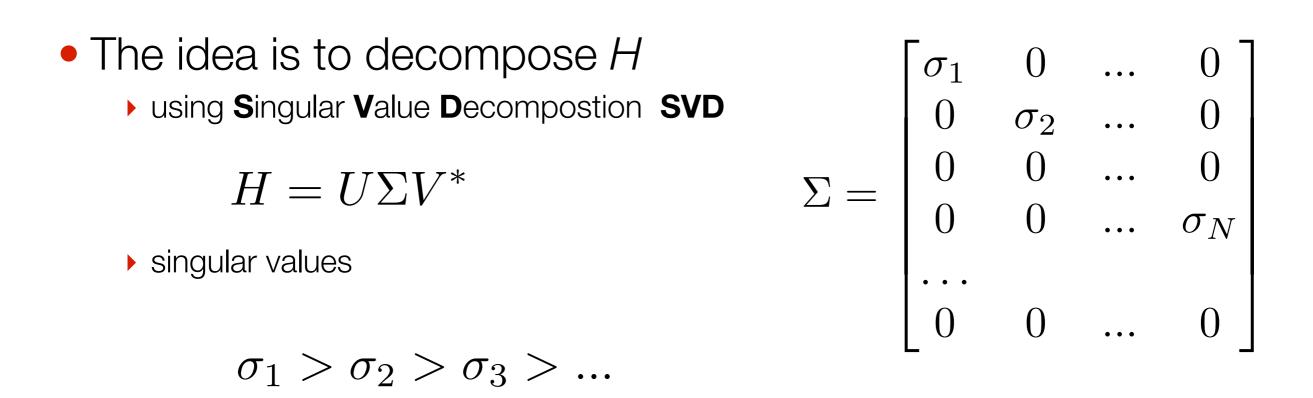
$S/N \sim \mathrm{invariant}$

Statistical treatment



Hankel matrix: Same terms on antidiagonals

Cadzow procedure



• we keep only the k largest singular values

- and reconstruct a denoised signal from the rank-reduced H matrix
- projection of H on a subspace

$$\tilde{H} = U\Sigma_k V^*$$

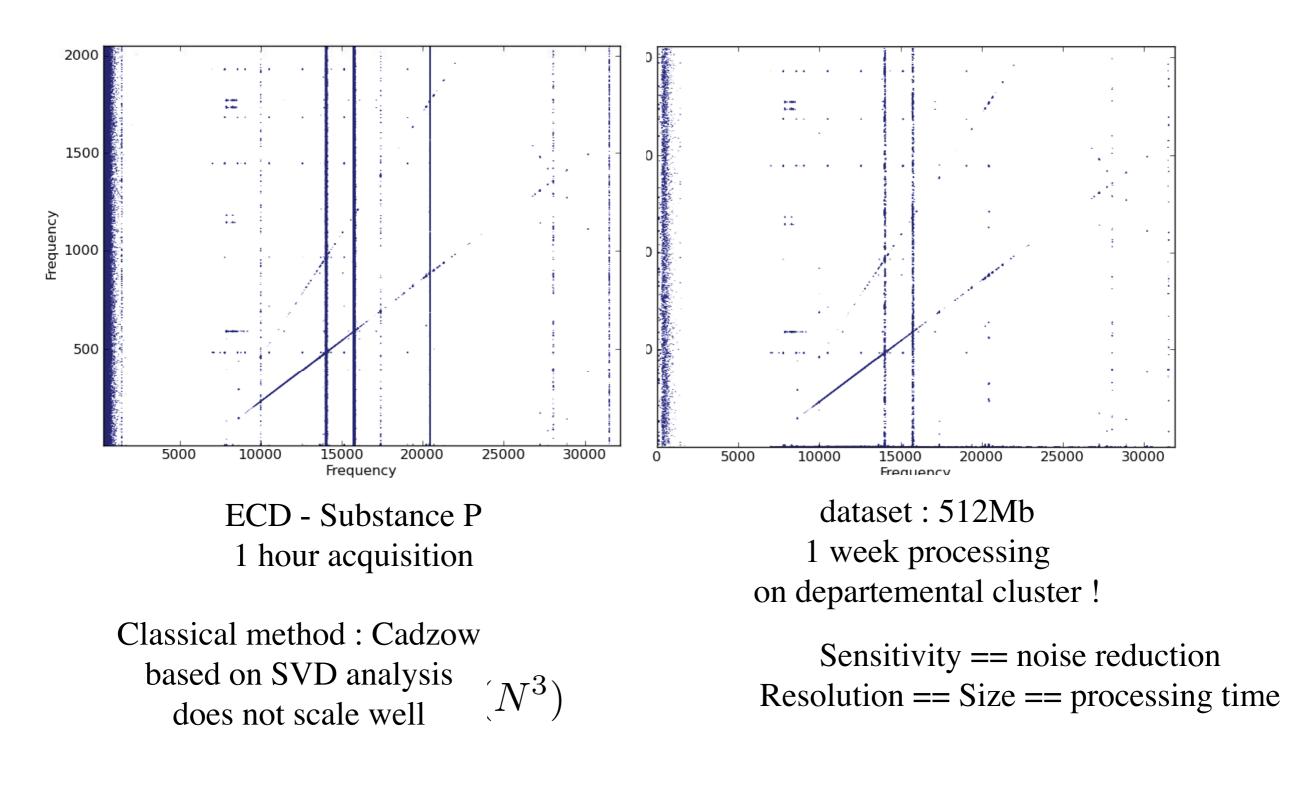
 $\tilde{H} = U_k U_k^* H$



then averaging on H antidiagonals

$$\tilde{x}_p = \langle \tilde{H}_{ij} \rangle_{i+j-1=p}$$

Cadzow, J.A. (1988) IEEE Trans. Acous. Speech Signal Proc., 36, 49-62.



Agthoven, M. A. V., Coutouly, M.-A., Rolando, C. & Delsuc, M.-A. Rapid Commun Mass Spectrom 25, 1609–1616 (2011).

Approximate by random sampling

combine several new mathematical ideas

use new developments linking between algebra and statistics

- Johnson Linderstrauss Lemma (1984)
- Compress Sensing approaches (Candès 2006, Donoho-Tanner 2007)

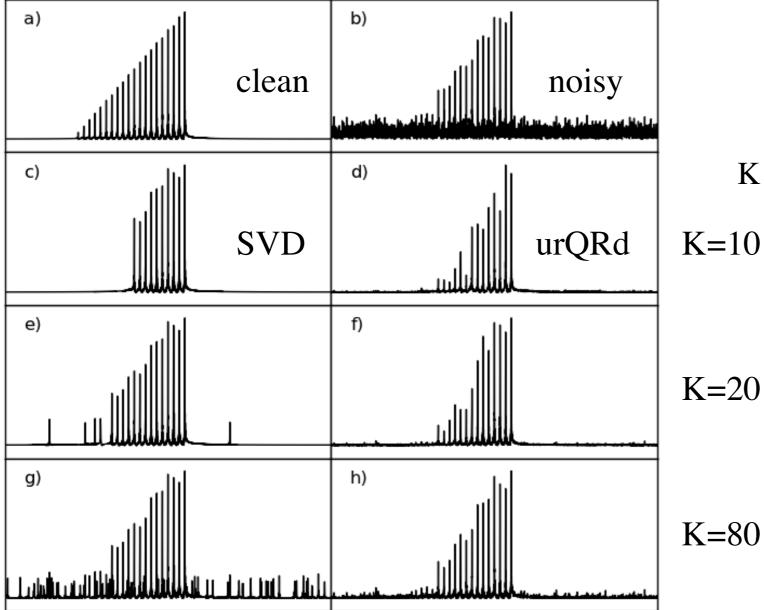
Apply matrix approximation rather than complete matrices
 Tygert, Martinsson (2007)

• \Rightarrow Estimate values rather than determining them

- SVD can then be replaced by QR decomposition (faster)
- precision and efficiency grows as the square root of the size hence efficient for Big Data

uncoiled random QR denoising : urQRd
 noise reduction from random sampling !

Example of urQRd on synthetic data



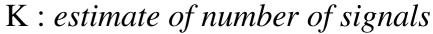
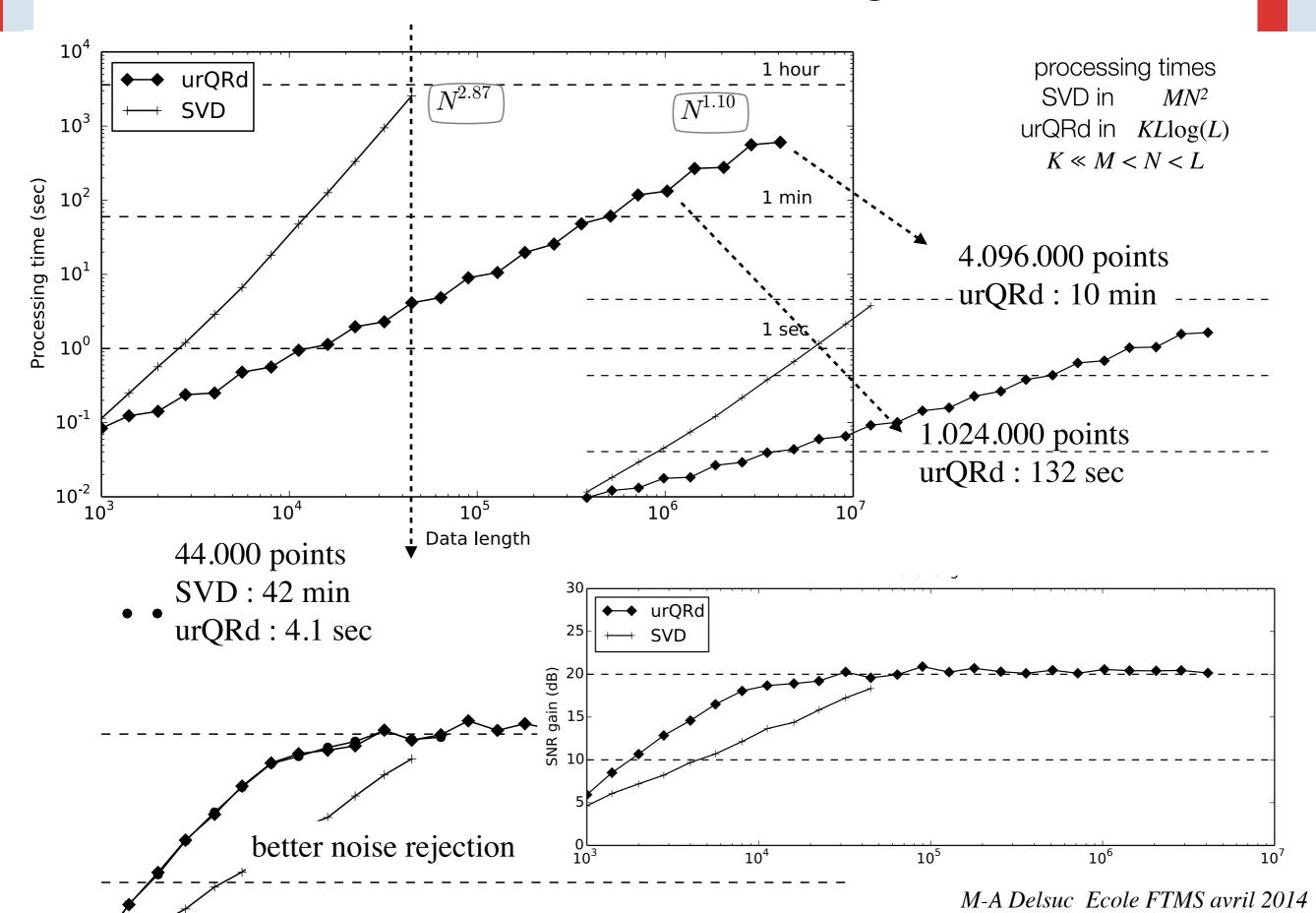


Fig. 1. Comparison of the the SNR gain afforded by the de-noising methods as a function of the rank. The computations are performed here on a synthetic complex 2000 points data-set containing 20 frequencies. a) Fourier Transform (FT) of the initial synthetic data-set composed of 20 lines of varying intensity. b) FT of the test data-set, with an added Gaussian white noise. SNR of the time-domain data-set is -0.14 dB. c-e-g) FT of the SVD processed of the synthetic data-set with with varying K. d-f-h) FT of the rQRd processed of the synthetic data-set with with varying K. c-d) rQRd and SVD processed of the synthetic data-set with K = 10 SNR gains : SVD 8.23 dB rQRd 2.91 dB, e-f) idem with K = 20 SVD 12.00 dB rQRd 5.13 dB. g-h) idem with K = 80 SVD 6.91 dB rQRd 9.95 dB.

much Faster - much Lighter



Applied to 1D FT-ICR data-set

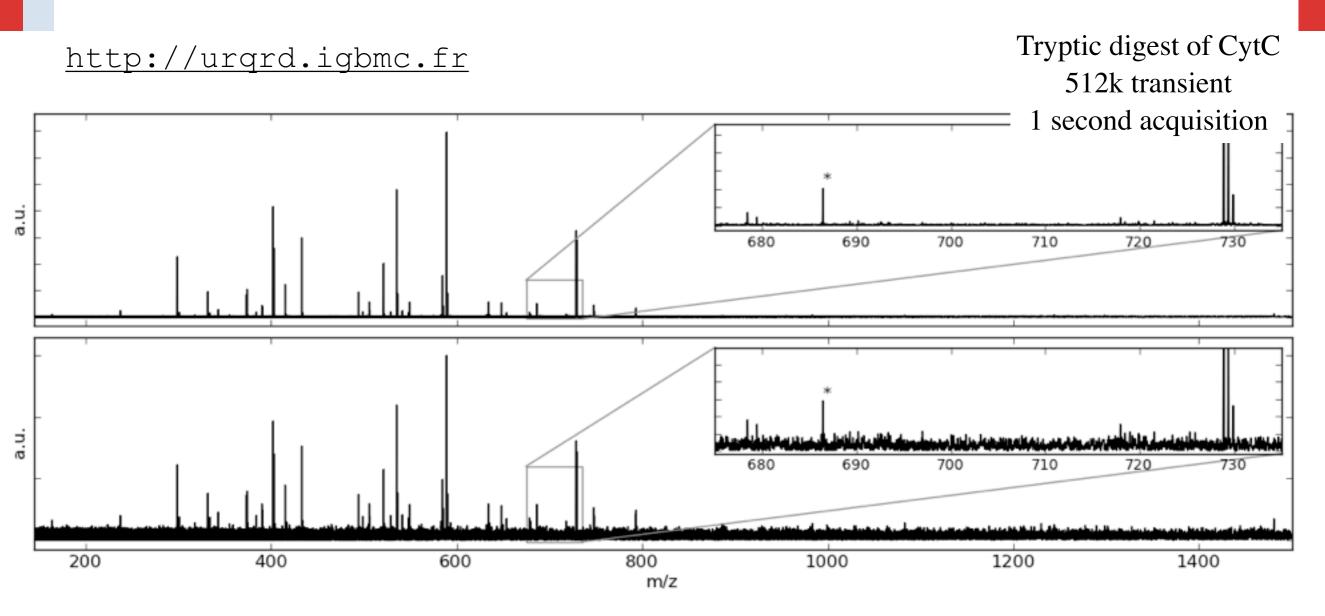
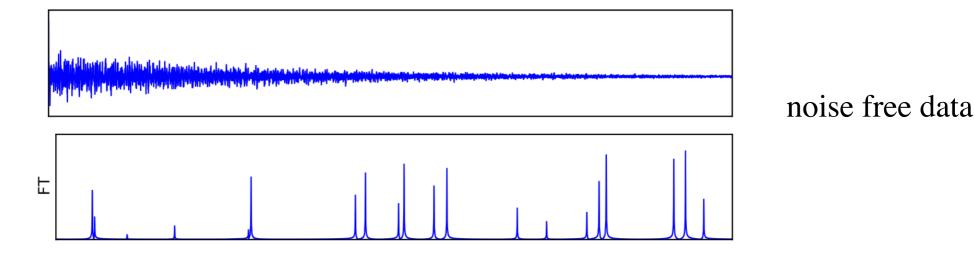


Fig. 4. Processing of a single-scan FT-ICR mass spectrum of a trypsin digest of Cytochrome C. *Bottom* original spectrum, SNR measured on the m/z 728.8388 peak is 24.0 dB. *Top* same spectrum after urQRd processing (K = 1000), SNR measured on the m/z 728.8388 peak is 40.7 dB. *inset*) the m/z 728.8388 peak corresponds to the TGQAPGFSTDANK²⁺ ion, m/z 678.3821 to YIPGTK+ and m/z 717.9012 to GEREDLIAYLKK²⁺. The peak labeled with a star at m/z=686.390, lacking isotopic structure, is likely to be an experimental artifact. The processed interferogram is 512k points, processed here with K = 1000.

SVD (~45 days512Go memory)urQRd25 min4Go memory

Chiron, L., van Agthoven, M. A., Kieffer, B., Rolando, C. & Delsuc, M.-A. *Proc Natl Acad Sci USA* **111**, 1385–1390 (2014).

Efficient on many kinds of noise



noisy data no noisy data scintillation additive noise-type : scintillation noise-type : additive rQRd filtered rQRd filtered noisy data noisy data missing points jitter noise-type : missing points noise-type : sampling rQRd filtered rQRd filtered

effect of urQRd denoising

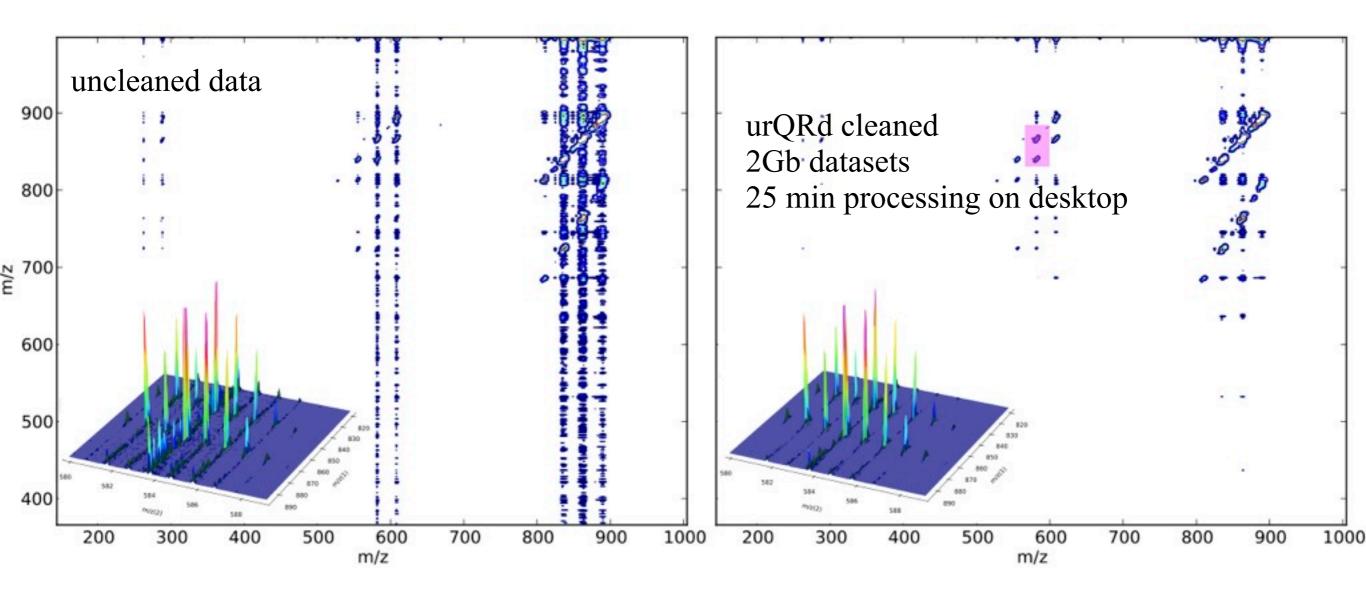
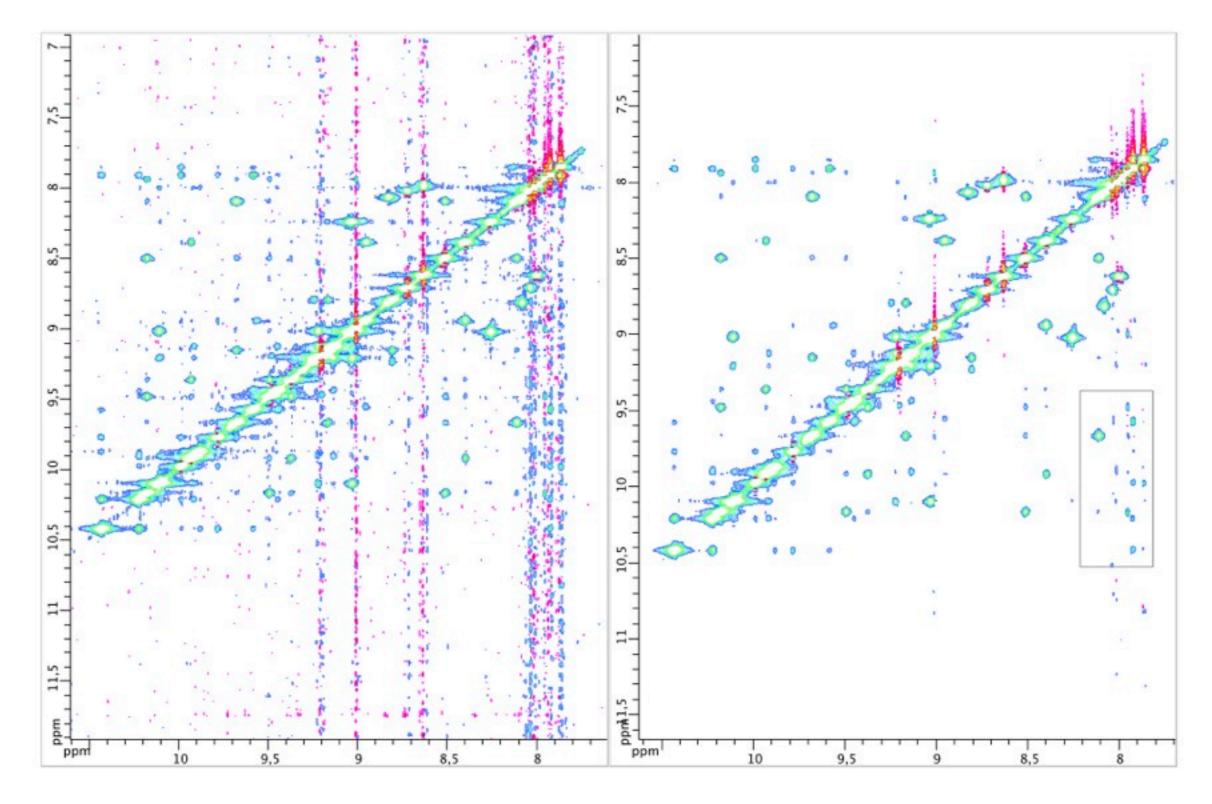
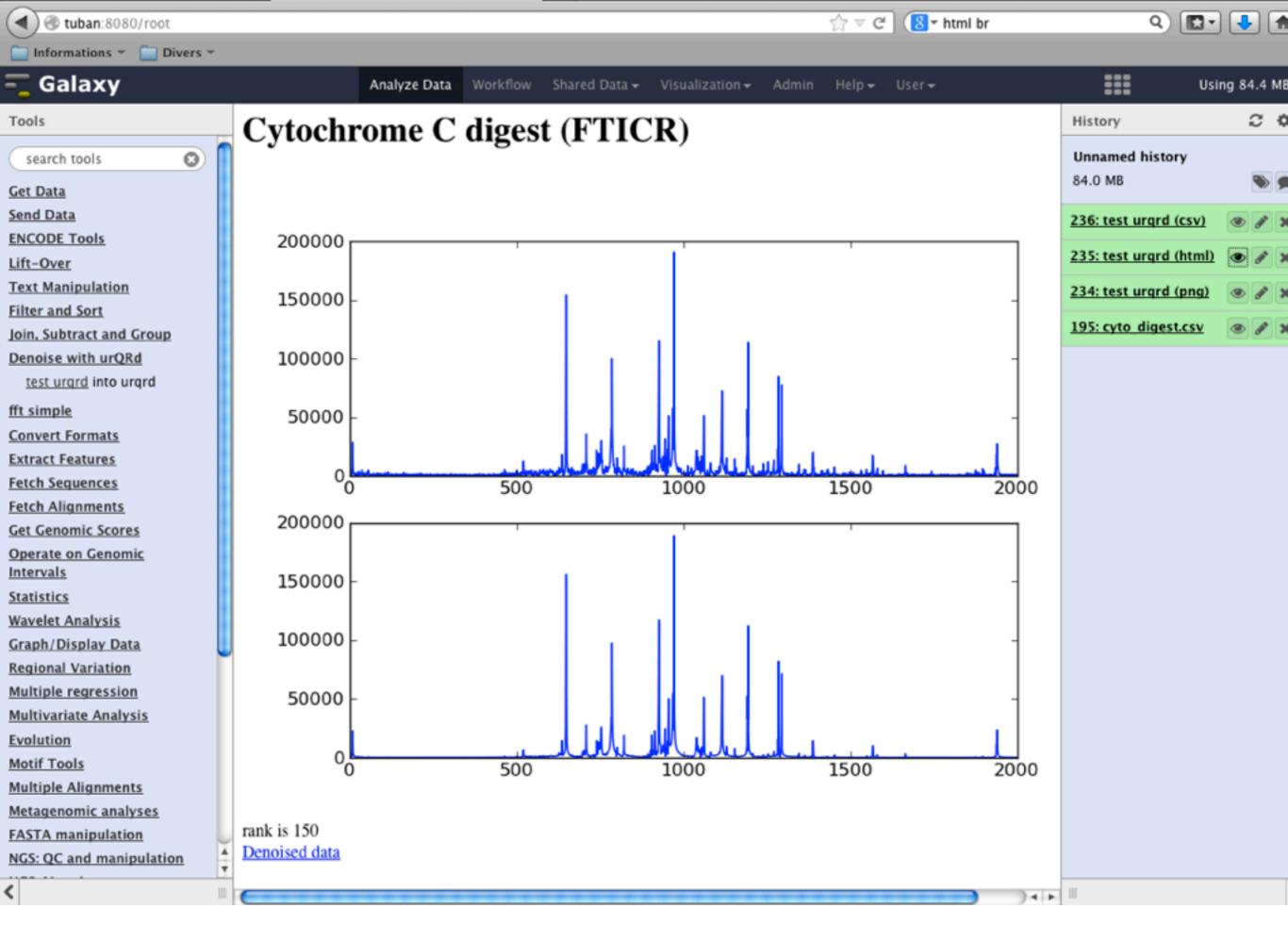


Fig. 5. 2D IRMPD FT-ICR MS spectrum of triacylglycerols extracted from human plasma showing strong scintillation noise. The data-set is $2k \times 128k$ points. *inset*) zoom on the pattern centered at m/z(F1) 845 and m/z(F2) 584 (highlighted in pink). The two groups of peaks give the isotopic patterns of lithiated TAG(16 :0/16 :0/18 :1) at m/z 839.7674 and lithiated TAG(16 :0/18:1) at m/z 865.7831 respectively losing a palmitic acid (MW 256.2396) and an oleic acid (MW 282.2553) in order to yield a lithiated diacylglycerol DAG(16:0/18:1) at m/z 583.5278(33). SNR was measured on the zoomed zone to 22.2 dB and 42.8 dB for the standard and de-noised datasets respectively.

Very efficient



Reduction of t1-noise on a 2D NOESY spectrum ~2minutes



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